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PHYSICAL
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L. L. Campbell

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PHYSICAL

LABORATORY MANUAL

FOR USE IN

SCHOOLS AND COLLEGES

BY

H. N. CHUTE, M.S.
AUTHOR OF "PRACTICAL PHYSICS," ETC., AND TEACHER OF PHYSICS
IN THE ANN ARBOR HIGH SCHOOL

REVISED EDITION

BOSTON, U.S.A
D. C. HEATH & CO., PUBLISHERS
1908

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Apr 8, 1936

Leslie Lyle Campbell

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Hence it would seem that the work of the laboratory should be largely quantitative in character, if all these advantages are to be gained by its introduction. Accordingly, the author has attempted to provide such a set of problems, judiciously distributed over the several divisions of physics, hoping thereby to touch the subject at most of its chief points, and at the same time keep within the mental grasp of students of secondary schools who are expected to pursue this subject.

Recognizing that there are many schools which have not adopted laboratory methods on account of the difficulty of obtaining apparatus of proper design at reasonable

expense, special pains have been taken in selecting problems that could be solved by the aid of simple and inexpensive apparatus. Many of the instruments are not beyond the skill of the average mechanic, and not a few are within the constructive ability of many a bright boy.

In the report of the Committee of Ten, appointed by the National Educational Association in 1892, occurs the following: "To give good instruction in the sciences requires of the teacher more work than to give good instruction in mathematics or the languages, and the sooner this fact is recognized by those who have the management of schools, the better for all concerned. The science teacher must regularly spend much time in collecting material, preparing experiments, and keeping collections in order; and this indispensable labor should be allowed for in programmes and salaries."

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Several valuable appendices conclude the volume, giving aid in the construction and in the repairing of apparatus, and furnishing a sufficiently complete list of the more important Physical Constants, accompanied by explanations when necessary.

H. N. C.

ANN ARBOR, MICHIGAN,
August, 1894.

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An experience of many years with students of varied preparation and ability has not changed the author's previously expressed opinion that the work of the laboratory should be quantitative in character, if the student is to receive from such work, in the largest measure, the benefits that should attend the study of physics. There seems to be a consensus of opinion among teachers that the order of topics might be changed to advantage; that with Sound and Light following Mechanics the grade is less steep than it is where these subjects follow Electricity. This, however, will not preclude the adoption of any other order that may be preferred. A few of the problems of the first edition have been omitted and new ones added, the better to adapt the book to the varied demands of teachers and of college requirements. Every problem has been carefully rewritten with such changes as the experience of the laboratory has shown to be desirable. The Tables of Constants have been carefully revised and extended. It is believed that they will now be found sufficiently extensive to meet all demands. A short table of Logarithms has been added in recognition of the fact that many teachers of physics believe that there is no valid reason why high school students should not be taught and required to perform all tedious arithmetical operations by the aid of logarithms.

The author's thanks are due the publishers for their generosity in furnishing so many cuts. These are faithful representations of actual pieces of apparatus that have been found very efficient, and at the same time not so expensive as to prohibit their introduction by schools of small means.

Although every effort has been made to eliminate errors, yet doubtless many remain, but it is believed that none will be found that will seriously detract from the usefulness of the book.

H. N. C.

ANN ARBOR, MICHIGAN,
October, 1903.

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Physics is too severe a study to be placed earlier than the fourth year in a high school curriculum. Students entering on this study should have their arithmetic, algebra, and plane geometry well in hand. The daily programme should be so planned, and students should be so classified, that each one in physics will have from two to three hours free during some one afternoon to devote to the laboratory. To divide up their time in the laboratory into short periods of an hour or less is to waste that time to a very large extent, and *work done under such conditions should never be dignified as laboratory practice.* Many of the experiments of physics cannot be completed in an hour, and it is not practical in the majority of cases to begin a problem one day with the expectation of completing it on another.

The didactic instruction should run parallel with the experimental, the former a little in advance of the latter. It will be found impossible to keep them together, neither is it desirable. Each student should have a *Text-book*, discussing the principles of physics, and a *Manual of Experiments*, describing the work expected of him in the laboratory. The custom of using up the time of a student in dictating to him matter that he can secure generally in much better form in a book, or of bringing him to the laboratory wholly ignorant as to what is expected of him till he gets there, is most pernicious, and can not be too severely condemned. It is such objectionable methods that have greatly retarded the development of physics teaching, and have contributed in no small measure to the dread with which students approach the subject.

9. Assignment of Work. — The problems that each student is expected to solve on his appointed day in the laboratory should be assigned to him several days in advance, in order that he may have an opportunity to study them thoroughly, and know what is expected of him. By proceeding in this way the student can accomplish a great deal more work in his allotted time, and do it very much better.

10. Selection of Work. — Teachers should feel perfectly free to omit any problem or set of problems that they choose. Although all problems are not equally important, yet circumstances will often determine which are to be omitted. This principle should govern the work: a few problems thoroughly studied will benefit the student more than many superficially gone over. The following list of problems has been adopted by the Physics Section of the Michigan Schoolmasters' Club, as best adapted to the facilities of the average school: 1. Hooke's Law. 2. Composition of Forces. 3. Laws of Accelerated Motion. 4. Laws of the Pendulum. 5. The Lever. 6. The Inclined Plane. 7. Pressure in Liquids. 8. The Siphon. 9. Boyle's Law. 10. Archimedes' Principle. 11. Density of Solids. 12. Density of Liquids. 13. Fixed Points on a Thermometer. 14. Coefficient of Expansion. 15. Specific Heat. 16. Latent Heat of Fusion of Water. 17. Latent Heat of Vaporization of Water. 18. Curve of Magnetization. 19. Mapping Magnetic Fields. 20. Simple Voltaic Cell. 21. Action of Electric Current on Magnetic Needle. 22. Electromotive Series. 23. Resistance of Conductors. 24. Resistance of Batteries. 25. Fall of Potential in Conductors. 26. Electromotive Force of a Cell. 27. The Tangent Galvanometer. 28. Velocity of Sound in Air. 29. Velocity of Sound in Solids. 30. Law of

Length for Strings. 31. Law of Diameter for Strings. 32. Law of Tension for Strings. 33. Photometry. 34. Law of Reflection of Light. 35. Measurement of the Angle of a Prism. 36. Image in a Plane Mirror. 37. Focal Length of a Concave Mirror. 38. Index of Refraction of Water. 39. Index of Refraction of Glass. 40. Focal Length of a Convex Lens. These are supposed to be preceded by five or six problems on simple measurements, involving the use of linear scale, calipers, and balance.

III. The Notebook.

11. Notebooks. — It is strongly recommended that the student keep two notebooks, the *temporary* and the *permanent.* One of the best forms of temporary notebooks is a blank book of the separable leaf type. It should be of good paper, unruled, and about 7 in. wide by 9 in. long. This book should be taken to the laboratory by the student, and in it he should record all his observations and measurements in neat tabular form, devoting, in general, two pages to each problem. The first of these should contain the number of the problem, a statement of its purpose, a list of the apparatus used, given both by name and by number, and a tabulated record of the observations and measurements. On the second page there should be sketches of the apparatus as set up for use, and any notes that it may occur to the student to make at the time, likely to assist him in writing up the problem after leaving the laboratory. This record leaf should be submitted by the student to the teacher in charge of the work and receive his approval before a new problem is entered upon, and should not be taken from the laboratory till such approval is received.

The permanent notebook should be more substantial in character; it should be of paper adapted to the use of ink;

and for convenience in drawing figures, tabulating observations, and the graphic representation of results, its pages should be ruled like cross-section paper, sometimes called "squared paper," and that preferably in the metric system. The size of the page should be about that of the temporary notebook. Into this book the student should carefully copy *in ink* the records of the temporary notebook after the following plan: Devote, in general, three pages to each problem. At the top of the *first* page write the *number* and the exact *purpose* of the problem. Follow this by *Apparatus*, about six lines from the top, giving under this heading a numbered list of the pieces used, together with drawings of the apparatus as set up for action. Near the bottom of this page, under the heading of *References*, list all the books consulted in completing the report on the problem, giving the page. At the top of the *second* page, under the heading *Results*, copy the tabulated record of the observations and measurements. The *Mathematical Calculations* should follow this record and complete the page. The *third* page should be devoted to a discussion of the results, unless the problem is of such a nature that a graphic representation of data is possible (15). In that case, devote the *third* page to the graphic representation and use the *fourth* page for the discussion.

The laboratory *should not be devoted to writing discussions of experiments, nor to making computations*, only so far as the results of these computations may be needed for immediate use; on the other hand, it should be given up wholly to *observational work, collecting data to be used and studied in preparing the final report on the problem.* No records should be made and taken from the laboratory on loose leaves of paper. Before leaving the laboratory the student should make out a copy of the data secured on a "data sheet" prepared for the purpose and leave it with the instructor. The instructor should consider the problem as unsolved when no "data sheet" is left by the student.

All measurements should be recorded exactly in the terms given by the scale or scales on the instrument. In a parallel column give the reduction to the form or denomination required. This is very important, and should be insisted on.

Avoid crowding the record; put down everything in such a way that others examining it will not need an interpreter. In recording measurements use the decimal notation, and never use common fractions unless the original instrumental readings should be in that form; in that case accompany them by their decimal equivalents. Record any peculiarity of the apparatus, especially if it tends to introduce inaccuracy into the work, and also record any measures taken to correct for such errors. It is better to record facts which further study may show to be irrelevant, than to find later that some necessary fact has been omitted from the record.

12. Mathematical Calculations.—These should be made on loose leaves of paper and copied into the notebook in equational form. *In all cases the results of the calculations must be transferred to the tabular record.* All calculations should be carefully revised and such "checks" applied as will eliminate from them all errors. Calculations as well as observations should show all the figures obtained that are trustworthy. There is an important difference between 4.2 and 4.20. The latter shows that the second decimal place has been determined and found to be zero, the former says nothing on this point.

An estimated quantity is always a doubtful one, and the accuracy of a measurement or a result is in no wise increased by an array of decimal figures, when, perhaps, all after the first are in doubt. Hence, it is just as well, in fact better, to omit all figures after the first doubtful one. But in dropping all such figures, the last retained figure should be increased by one unit if the part dropped exceeds 5. For example, in the number 2.1**5**53, the shaded **5** is the first doubtful figure; then, under the rule given, the number would be written 2.16; if the number were 2.1**5**5, then it would be written 2.15 with no increase in the last figure, because the dropped part does not exceed 5.

When multiplying numbers together, notice how far the doubtful figure of the factor affects the results and drop all figures after the first doubtful one. For example, if the product of 2.31**4** by 3.21**3** is required, the full calculation is as follows:—

$$\begin{array}{r}
2.31\mathbf{4}\\
3.21\mathbf{3}\\ \hline
\mathbf{6942}\\
231\mathbf{4}\\
462\mathbf{8}\\
694\mathbf{2}\\ \hline
7.434\mathbf{882}
\end{array}$$

It appears from an inspection of the work that the last four figures of the product are in doubt and that the product expressed as 7.435 is equally as exact as the whole six places.

The labor of obtaining this product may be considerably lessened by proceeding as follows:—

$$\begin{array}{r}
2.31\mathbf{4}\\
\mathbf{3}123\\ \hline
694\mathbf{2}\\
463\\
23\\
\mathbf{7}\\ \hline
7.43\mathbf{5}
\end{array}$$

Write the multiplier beneath the multiplicand, with the figures in reverse order. Then form the partial products in the ordinary way, always beginning by multiplying each figure into the one above it, increasing the product by what there would be to carry if the product into the next figure to the right had been found.

In the example given, it will be seen that the result gives all the figures that are trustworthy, and omits those that are doubtful.

The same principle applies in division, as appears from the following example:—

31.2**5**)581.93**5**(18.6**2**	31.2**5**)581.93**5**(18.6**2**
312**5**	312**5**
269**4**3	269**4**
2500**0**	2500
19**435**	19**4**
18750	18**7**
6850	**7**
6250	**6**
600	**1**

The heavy figures, as before, show the effect of having the last figure of both dividend and divisor in doubt. The fourth figure of the quotient is in doubt, and the division should end at that stage. In the right-hand operation the same quotient is obtained by omitting to bring down a new figure for each partial division and offsetting it by dropping one figure from the divisor in forming each partial product.

The application of these contractions effects quite a large saving of time in making the calculations attending physical observations, without any detraction from the accuracy of the conclusions reached.

Another great saving of time can be secured by the use of Logarithmic Tables. A set of four-place tables will be found in the Appendix.

13. The Weekly Report.—When the work assigned for the week is completed by the student and written up in the permanent notebook, it should be submitted to the instructor for examination and criticism. From an examination of the preceding articles it will appear that this report should be comprised of the following parts: (1) *A Statement of the Specific Object of the Problem.* (2) *The Apparatus used,* each piece being designated in such a manner that subsequent identification will be possible. (3) *Drawings* that show the plan of the instruments, made accurately to scale, when possible, and free-hand sketches showing the set-up of the apparatus when the experiment is in progress. For example, the drawing in connection with the use of the vernier caliper should show the graduation of the vernier, the scales being accurately drawn, but under a problem in electrical measurements it should show the manner of connecting together the instruments comprised in the electrical circuit, rather than pictures of the apparatus used. (4) *References to Books consulted in Preparing the Report,* giving title and page. (5) *The Record of the Observations* made in the laboratory, accompanied by such reductions and transformations as were made subsequently. (6) *The Mathematical Calculations.* (7) *The Discussion of the Results Obtained.* This discussion should be written in the quiet of the study room after all the facts observed have been carefully considered. It should first be written out on loose paper, criticised and corrected by the student himself, and when put in the best shape that he is capable of giving it, it should be copied in ink on a sheet of note paper and placed in the notebook adjacent the page reserved for it. After it has been examined by the instructor, the student should, after correcting the errors marked, copy it in the notebook. This discussion, when completed, should comprise a statement of the physical and mathematical principles involved in the problem, and answer all questions proposed under *Directions* and *Remarks* in the Manual. It should consider the nature of the errors that enter into the work, pointing out those that are caused by the imperfections of the apparatus, and those that are due to the inexperience and perhaps carelessness of the observer. It should point out what means were taken to reduce or eliminate errors. In discussing errors and the effects produced by them, do not tell what you may have done, but rather what you undoubtedly did do. Whenever correct values or accepted values are known, these should be recorded, compared with those obtained, the per cent. of error calculated and given a conspicuous place at the end of the tabulated record. Whenever the results obtained admit of a physical interpretation or statement as a law, this should be made a prominent feature of the discussion; and whenever the data are used in plotting a curve (15), the inferences to be derived from the curve should also be carefully stated.

Every part of the report should be written with pains, equally from a mechanical, a scientific, and a grammatical standpoint. Slovenly, careless work should be rejected, and nothing but the best that the student is capable of should be accepted. The keeping of a neat notebook should be encouraged in every way. It is time well spent, and the effect on the student is most wholesome.

14. Errors in Observing. — All observations are liable to be affected by two kinds of errors, which may be distinguished as *Accidental* and *Systematic*. The former are due to the imperfections of our senses and of the apparatus employed, and are seen when several measurements have been made of the same magnitude by the results being characterized by small differences. Any one of these values is just as likely to be greater than the true value as it is to be less; and of the entire number of measurements, those in excess and those in deficit will probably be equally numerous. The arithmetical mean of all the results obtained is likely, therefore, to be more nearly correct than any one result by itself. No amount of repetition, however, will eliminate the systematic errors. They can be obviated only by first finding out their causes and taking measures to remove these causes or determine the magnitude of their effects. Among such errors may be enumerated air currents, temperature changes, barometer changes, errors in the graduation of the apparatus, etc.

In a series of observations, if one or more are observed to differ widely from the arithmetical mean of the entire set, it is fair to conclude that these are too much in error to use, and the result to be adopted should be the mean obtained of the observations remaining after rejecting those so widely at variance. Those not used in obtaining the mean should be inclosed in brackets. When the final result sought is to be derived by calculation from the measurements made, it will often happen that errors in some of the quantities will not produce so serious an effect as errors in some others. Hence it appears that some observations must be made with greater care than others.

On account of the facility with which errors are made, it is advisable to vary as much as possible the conditions under which the experiment is conducted, to the end that new combinations may arise and the sources of error be detected. "Check methods" should be used whenever possible; even if they are not calculated to give close results, they will assist in detecting errors. The aim should be to secure the highest degree of accuracy attainable with the quality of apparatus used and the amount of time that can be devoted to the problem.

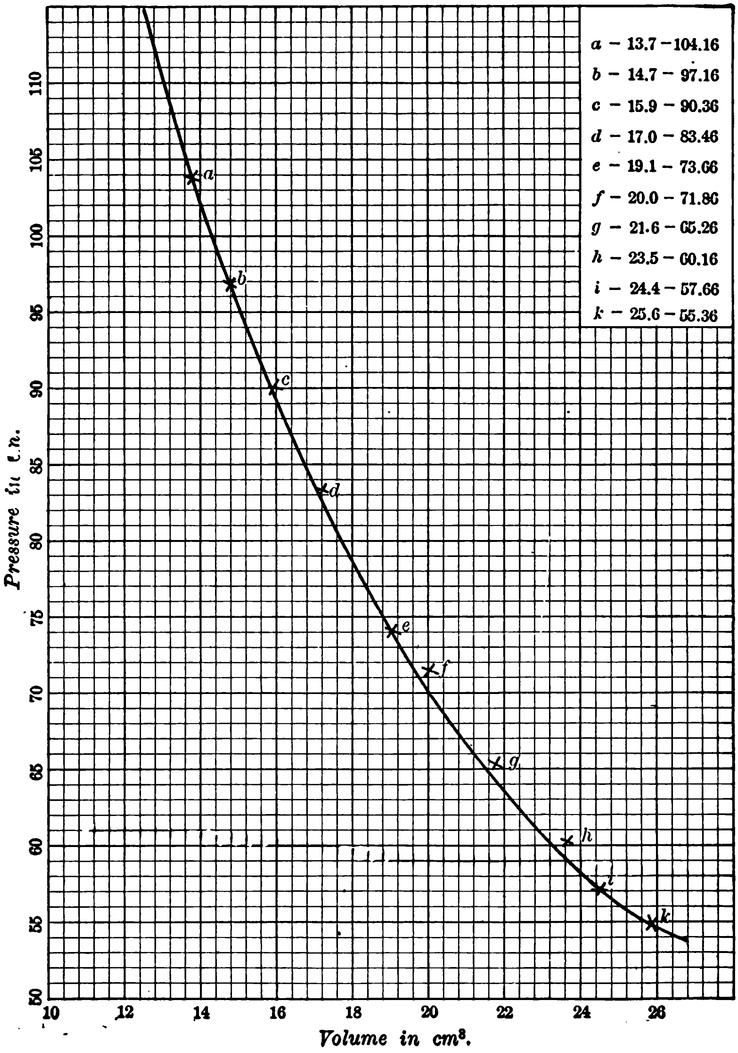
15. Graphic Representation of Data. — This method is employed to show to the eye the relation between two quantities which are so connected that any change in the value of one is attended by a change in the other; as, for instance, the height to which water rises in a capillary tube is greater as the diameter of the tube is less, or the upward pressure of water on a submerged area is proportional to the vertical depth of that area below the surface of the water. It consists in representing by a line the data obtained by the observer, the shape of the line or *graph* often indicating the form of the law that expresses the relation between the interdependent facts.

To construct the curve, procure a sheet of cross-section paper of the size of a page of the notebook. This will not be necessary if the paper of the notebook has cross-sectional ruling. Cross-section paper is paper divided into small squares by two sets of parallel lines cutting each other at right angles. These squares may be 1 mm. or 2 mm., or 0.1 in. on each side. For convenience, when the paper is metrically ruled, every centimeter line is ruled darker, and when the tenth of an inch is the unit, every half-inch line is darker. Select some suitable scale to be used in representing the quantities, the chief consideration being that the scale unit should be such as will confine the curve to the page, and yet extend it practically over the whole of it. In some instances the sum of the sides of several of the squares may constitute the linear unit; in others the side of a square may represent several units. It is not necessary that the same unit be chosen for both magnitudes involved. In general, it is best to make the value of the scale division correspond to the least quantity that can be measured. The scales chosen should be marked along the adjacent sides of the page, and only whole numbers should be used. The corner of some one of the squares should be selected as the *Origin* and marked *O*.

This is usually taken near the lower left-hand corner of the page, although sometimes it is advisable to place the *O* off the page. The horizontal line through the origin is called the *Axis of X*, and measurements made on it are called *Abscissæ*. The vertical line through the origin is the *Axis of Y*, and measurements made on it are called *Ordinates*. Either set of the related data may be considered as abscissæ, the other being used as ordinates, and the problem is to locate a point on the paper for each pair of quantities, the quantities being considered in the nature of latitude and longitude of these points. Measure off on the axis of *X* a distance that represents the numerical value of the first quantity; and then on the axis of *Y* a distance representing the quantity dependent on it. The point sought is at the intersection of vertical and horizontal lines respectively through these points on the axes, and is marked by a small oblique cross, ×. Positive quantities are always measured to the right or upward from the origin, and negative quantities in the opposite direction. In like manner, locate all the points, and then through them trace as smooth and regular a curve as possible. The simpler the line passing through these points, the less complex the law connecting the quantities is likely to be.

A table of the data from which the curve is plotted should accompany the figure, the values assigned to a scale division on the horizontal and vertical axes should be marked along these axes, and the nature of the magnitudes represented should also be marked along the axes.

On account of errors in the data, it is not likely that all points will lie exactly on a smooth curve. Hence, the curve often indicates where mistakes exist. It is also of service in getting approximate values for the dependent quantities at stages in the phenomena where no observations were taken.


a – 13.7 – 104.16
b – 14.7 – 97.16
c – 15.9 – 90.36
d – 17.0 – 83.46
e – 19.1 – 73.66
f – 20.0 – 71.86
g – 21.6 – 65.26
h – 23.5 – 60.16
i – 24.4 – 57.66
k – 25.6 – 55.36
a
b
c
d
e
f
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h
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75
70
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50
10
12
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16
18
20
22
24
26
Pressure in c.m.
Volume in cm³.

FIG. 1.

To illustrate the process, let it be required to plot a curve from the following data: —

VOL. OF AIR	PRESSURE	VOL. OF AIR	PRESSURE
cm.^{3}	cm.	cm.^{3}	cm.
a. 13.7	104.16	*f.* 20.0	71.86
b. 14.7	97.16	*g.* 21.6	65.26
c. 15.9	90.36	*h.* 23.5	60.16
d. 17.0	83.46	*i.* 24.4	57.66
e. 19.1	73.66	*k.* 25.6	55.36

In order that the curve may occupy the middle portion of the page, the origin is not placed on it. Hence, let the horizontal scale begin at 10 (Fig. 1), and each centimeter represent 2 cm.^{3} of volume. Also, let the vertical scale begin at 50 and each centimeter represent 5 cm. of pressure. Then on the horizontal scale each small division will stand for 0.4 cm.^{3}, and on the vertical scale each small division will stand for 1 cm. The point *a* is located by measuring to the right on the horizontal scale 13.7 units, and then upward on the vertical scale 104.16 units. In both cases the decimal parts are estimated by the eye. Each point is located in like manner, and a smooth curve drawn so as to include the greatest number. It will be noticed that the points *f*, *g*, and *h* are without the curve. This indicates that probably some error was made in determining the data for them.

CHAPTER I.

SIMPLE MEASUREMENTS.

I. Measurements of Length, Area, and Volume.

16. Measurement in General. — The process of measurement consists in comparing the magnitude, either *directly* or *indirectly*, with some assumed standard in order to ascertain the numerical relation between them. The accuracy of the results obtained will depend upon that of the standard, upon the methods adopted to insure close comparisons, and upon the carefulness of the experimenter. **A direct method** is one in which there is an actual application of the standard unit of measure to the thing to be measured. **An indirect method** is one in which other quantities than the one stated in the problem are measured, and the values sought are deduced from these data by a mathematical process. A simple illustration of this method is seen in the problem of finding the area of a triangular figure by measuring the length of its sides and computing the area by certain geometrical relations (27).

17. Units Employed. — In the measurement of length there are two standard units in general use, the **meter**, and the **yard**. The legal definition of the former is the length of a platinum-iridium bar at 0° Centigrade, that is kept in the National Bureau of Standards at Washington, D.C. It is a copy of the Mètre des Archives, in the possession of the French government. The **yard** may be defined as $\frac{3600}{3937}$ meter. It is divided into 3 equal parts, called **feet**, and each foot into 12 equal parts, called **inches**. The inch is subdivided, either decimally or into 8, 16, 32, or 64 equal parts. The meter is subdivided decimally into 10 decimeters, 100 centimeters, or 1000 millimeters. In writing quantities in the metric system, the decimal notation should be used; instead of 123 m. 4 dm. 5 cm. 6 mm., it should be 123.456 m., or 12345.6 cm.

The units of area are derived from those of length, and are usually squares whose sides are some one of the units of length, as the **square inch** (sq. in.), **square foot** (sq. ft.), **square millimeter** (mm.^{2}), and **square centimeter** (cm.^{2}), where the sides are 1 in., 1 ft., 1 mm., and 1 cm., respectively.

The units of volume are also derived from those of length, and are usually cubes whose edges are some one of the units of length, as the **cubic inch** (cu. in.), where the edge is 1 inch; the **liter** (l.), where the edge is 1 dm. or 10 cm.; the **cubic centimeter** (cm.^{3}), where the edge is 1 cm., etc. The **U.S. gallon** is a unit of volume of no fixed shape, the only condition being that it shall contain 231 cu. in. The **liter**, likewise, may have any desired shape, provided it shall contain 1000 cm.^{3}.

18. Relation of the Metric to the English Units.—The meter is equal to 39.37 in. Hence, if any linear quantity expressed in inches be divided by 39.37, the quotient will be meters; and if any such quantity expressed in meters be multiplied by 39.37, the product will be inches. The inch is equivalent to 2.54 cm., or 25.4 mm.; the square inch to 6.452 cm.^{2}; the cubic inch to 16.387 cm.^{3}. For other reduction factors see Table XXII of the Appendix.

19. Problem. — *To measure the length of a table, estimating the tenths of the smallest scale division.*

Apparatus. — A **meter rod** (Fig. 2), a **needle,** and a flat **block** of **wood** having at least one square edge. The meter rod, as furnished by the American Metric Bureau, Boston, Mass., is a wooden bar, of rectangular cross-section, graduated to millimeters on one face, and to the tenths of an inch on the opposite one.

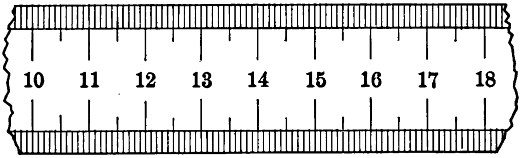

FIG. 2.

Directions. — Apply the scale directly to the top of the table and parallel to the edge. Hold the wooden block firmly against the edge of the end of the table, and its face will mark where the measurement is to begin. If the table is longer than the meter rod, mark the end of the rod by sticking the needle in the table, and then slide the rod along to a position wherein one end is exactly at the point marked by the needle, and this second position of the rod forms a straight line with the first position. Proceed in this way till the rod projects over the other end of the table. Now, by placing the block against this end with its square edge against the face of the meter rod, the length can be obtained to within a millimeter; and by an eye-division of the millimeter to tenths, the length can be found to tenths of a millimeter. A small pocket magnifier should be used to assist in estimating the tenths, and also in placing the end of the meter rod or any specified point of the rod in coincidence with the point marked by the needle.

In using the English side of the scale, on account of the awkward fraction, .37 in., it is better to use but part of the rod, say, 30 in.; that is, use the rod as if it were cut off at that division.

The measurements should be repeated several times in each system and the arithmetical mean of each set taken. Reduce the mean value in inches to centimeters, and find the difference of the two results. Since the last figure of the measurements is in doubt, the mean should not be calculated farther than tenths of a millimeter (12).

Remarks. — This method of measurement is a direct one. It is evidently marked by several important sources of error. The chief one is that known as **parallax**, a shifting in the apparent position of a scale division with a change in the position of the eye. To reduce this error to the smallest quantity, the divisions of the scale must be brought as close as possible to the object measured. Figure 3 illustrates an incorrect position of the scale, and makes it clear that the reading will be affected by changing

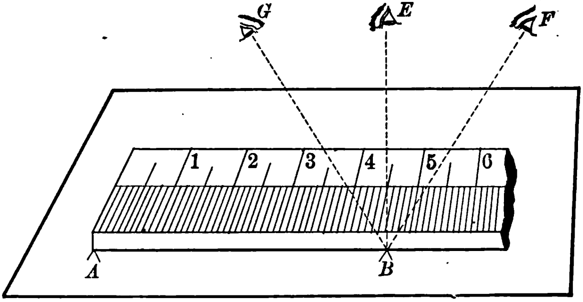

FIG. 3.

the eye from *G* to *F*. Figure 4 illustrates the correct position of the scale, in which the divisions are brought as close as possible to the surface of the table. Another error is due to the measurement not being made along a perfectly straight line and parallel to the edge of the table. A third source of error is likely to be an imperfect meter rod. Wooden linear measures are apt to warp (What effect would that have on the accuracy?), and, if much used, to wear at the ends. It is better never to make any linear measurement from the end of a scale, not only on account of the error caused by the wearing off of the end, but also on account of the difficulty of setting a mark exactly at the end. Any division line on a scale can be marked off on a line far more accurately than the end can, and any scale mark can be placed at a given point on a line with greater accuracy than the end can be so placed.

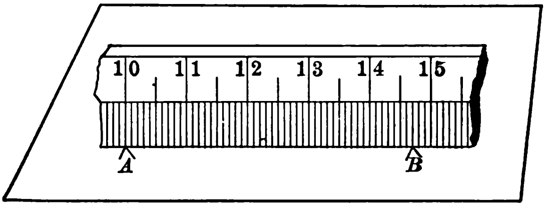

FIG. 4.

Form of Record. — The following example illustrates the manner of recording in the notebook the measurements made: —

First measurement . . .	212.90 cm.	83.75 in.
Second “ . . .	212.85 cm.	83.80 in.
Third “ . . .	212.88 cm.	83.78 in.
Fourth “ . . .	212.87 cm.	83.79 in.
Mean	212.87 cm.	83.78 in.

83.78 in. = 212.80 cm.
Difference = 0.07 cm.

(QUERIES. — Why do the successive measurements differ? What reasons can be assigned for the fact that the mean of the measurements made with the English side of the rod differs from that made with the metric side? Answer these in the discussion.)

20. Problem. — *To measure the length of a straight line drawn on a page of the notebook.*

Apparatus. — A pair of **dividers** (Figs. 5 and 6) and a **diagonal scale** (Fig. 7). The former is merely a pair of compasses with fine steel points. The latter is a scale of equal parts, having eleven equidistant parallel lines running lengthwise of the scale, and divided into a number of equal parts by perpendicular lines. The distance between two consecutive perpendicular lines is the unit of the scale, and is, usually, either ½ in., 1 in., 1 cm., or 2 cm. This unit, at one end of the scale, on both the first and the eleventh horizontal line, is subdivided into ten equal parts, and these points of division are joined by diagonal lines. These slanting lines are $\frac{1}{10}$ of the scale unit apart; and each one intersects a horizontal line at a point nearer to or farther from any perpendicular line than the adjacent line by $\frac{1}{100}$ of the scale

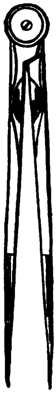

FIG. 5.

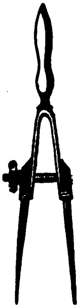

FIG. 6.

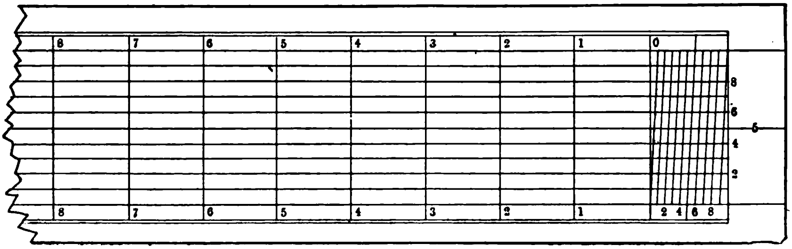

FIG. 7.

unit. The diagonal scale, therefore, reads to hundredths of the scale unit.

Directions. — Open the dividers and place its points exactly on the ends of the line. A pocket magnifier will greatly aid in doing this accurately. Now place one point of the dividers on one of the divisions 1, 2, 3, etc., of the scale, selecting such a one as will cause the other point of the dividers to fall either at 0, or between 0 and the vertical line bounding the scale. If, for example, by placing the left-hand point on the perpendicular 5, the right-hand point falls on 0, then the length is 5 units; but if the right-hand point falls on the diagonal line 3, then the reading is 5.3 units. If, however, the right-hand point falls between 3 and 4, then move the dividers toward the opposite edge of the scale, keeping the line of the points parallel to the horizontal lines of the scale, and at the same time the left-hand point on the perpendicular line 5, until the right-hand point fits exactly on the intersection of a diagonal line with a horizontal one. If this, for example, occurs at the intersection of line 3 with the horizontal line 7, then the reading is 5.37 units. The scale reading multiplied by the value of the scale unit, expressed either in inches or centimeters, will give the distance sought. After a measurement has been made and recorded, *close the dividers and repeat the work.* Several independent measurements should be made and the arithmetical mean calculated. Since the last figure is in doubt, the mean should not be calculated beyond the second place (12).

Remarks. — Hold the dividers at *an oblique angle* in applying them either to the paper or to the scale. *Avoid pressure* likely to indent the surface. The hinge of the dividers should work smoothly but not too easily. Hold the dividers so as to avoid closing them while passing from the line to the scale. Study the scale so that you may know what is the unit used in constructing it. *Study every feature of the problem so as to be able to point out in the discussion the most probable and serious sources of error entering into the method of solution.* This method of measuring a linear magnitude is an indirect one (16), and may be employed whenever a scale can not be applied to the magnitude to be measured.

Form of Record.— The following example illustrates the manner of recording the measurements: —

First measurement	5.37
Second "	5.38
Third "	5.36
Fourth "	5.37
Fifth "	5.38
Mean	5.37
Value of scale unit	1 cm.
Length of line	5.37 cm.

21. Problem. — *To measure the diameter of a ball.*

Apparatus. — Two **square-cut blocks** of wood and a **meter rod.** The angles of the blocks must be square, and their thickness must not be less than the semidiameter of the ball. The edges of the blocks must be sharp lines.

Directions. — Press the blocks firmly against the edge of the meter rod. Place the ball (Fig. 8) between the inner faces of the blocks, and as close as possible to the meter rod. With the blocks touching the ball, read on the scale

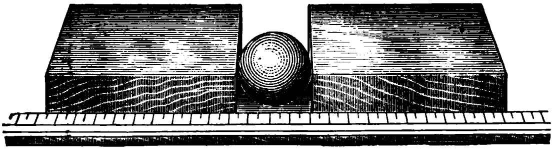
FIG. 8.

the exact position of the inner faces, using a magnifier to estimate the tenths of a millimeter. The difference between these readings will be the diameter of the ball.

Remarks. — It is preferable to place one block exactly on the division 10 of the meter rod. Measure the ball in a number of different positions. How would the results be affected if the blocks are not square-cornered? Show by a figure in the discussion. How could a correct result be obtained with such blocks? Test the blocks by placing them together against the meter rod, with the faces in contact, and see if the contact is perfect throughout their entire length. How would it affect the results if the thickness of each block is less than the semidiameter of the ball? Show by a figure. What are the chief sources of error? Which are personal?

Form of Record. — The following example illustrates the manner of recording the measurements: —

	POSITION OF FIRST BLOCK	POSITION OF SECOND BLOCK	DIAMETER
	cm.	cm.	cm.
First measurement . . .	10.00	14.25	4.25
Second " . . .	10.00	14.28	4.28
Third " . . .	10 00	14.26	4 26
Fourth " . . .	10.00	14 25	4.25
Fifth " . . .	10.00	14.24	4.24
Mean			4.26

22. Problem. — *To find the gauge number and the diameter of a wire.*

Apparatus. — The **micrometer caliper** (Fig. 9) and the **wire gauge** (Fig. 10). The micrometer caliper consists of a stout, curved steel frame carrying two screws, *B* and *C*, the latter of which moves freely by means of the head, *D*. The pitch of the screw, *C*, is obtained by observing the value of the spaces on the linear scale, *A*, and how many turns of the head are necessary to carry the edge of the sleeve, *D*, exactly over one of these divisions. If, for example, the divisions on the scale, *A*, are millimeters, and one turn of *D* carries its edge exactly over one of these divisions, then the pitch of the screw is a millimeter. If, as is the case with some instruments, two turns of *D* are necessary to move the edge over one division, then the pitch is $\frac{1}{2}$ mm. The circular scale on *D* usually consists of 25 divisions when the pitch of the screw is $\frac{1}{40}$th of an inch, 20 divisions when $\frac{1}{50}$th of an inch, 50 divisions when $\frac{1}{2}$ mm., and 100 divisions when 1 mm. Hence, one division on *D* represents $\frac{1}{25}\times\frac{1}{40}=.001$ in. in the first case, $\frac{1}{20}\times\frac{1}{50}=.001$ in. in the second case, $\frac{1}{2}\times\frac{1}{50}=.01$ mm. in the third case, $\frac{1}{100}\times 1=.01$ mm. in the fourth

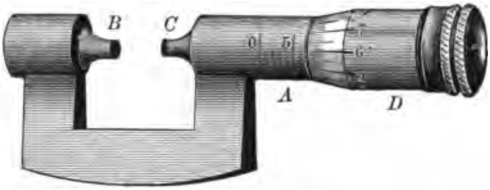

FIG. 9.

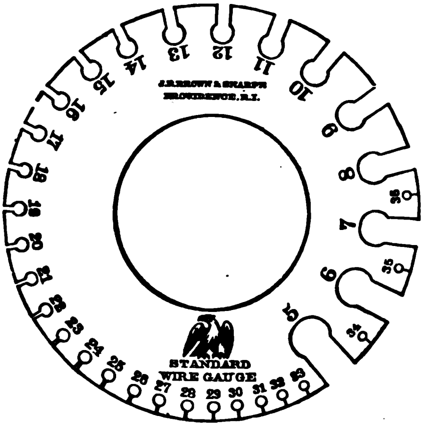

FIG. 10.

case. In the third case it should be noticed that since the limb readings are in millimeters, the head readings are to be increased by 50 whenever the partial millimeter space on the limb exceeds a half space.

The wire gauge is a circular disk of steel, with square-faced slots cut in the circumference. These slots vary in width and are numbered on the scale from 0 to 40. There are several wire gauges in use; the one shown in Fig. 10 is the Standard American, otherwise known as the Brown and Sharpe Gauge.

Directions. — To ascertain the number of the wire, find which slot in the rim of the gauge will just permit the wire to pass through, noting the number adjacent the slot. To measure the diameter, place the wire between the flat faces of the screws, *C* and *B*, of the micrometer and turn *D* till contact is secured. In some instruments, the sense of touch must be depended on for this; in others, when a certain pressure is secured, the head, *D*, slips around without advancing the screw, *C*. Now observe how many complete spaces are exposed on the limb, *A*; then with a magnifier ascertain what division of the circular scale on *D* coincides with the line running lengthwise of the cylinder carrying the scale, *A*. Express this head reading decimally and add it to the reading on *A*. Now remove the wire and turn *D* till *C* and *B* are in contact. Take the reading as before. This is called the zero reading. The difference between these readings is the diameter.

Remarks. — If the instrument is not constructed so that the head, *D*, slips when *C* touches *B*, care must be taken that no undue pressure is exerted, as it might spring the instrument. Often the wire is curved; then it is evident that the plane of the curve should be placed perpendicular to the axis of the screw (Why?). If no circular division of

D fits exactly on the line on *A*, then estimate in tenths of the head reading the position of this line.

Form of Record. — The following example illustrates the manner of tabulating the measurements: —

WIRE NO. 8 (B. AND S.).

	ZERO READING	FINAL READING	DIAMETER
First measurement . .	0.014 mm.	3.245 mm.	
Second " . .	0.015 mm.	3.244 mm.	
Third " . .	0.014 mm.	3.250 mm.	
Fourth " . .	0.014 mm.	3.247 mm.	
Fifth " . .	0.015 mm.	3.246 mm.	
Mean	0.014 mm.	3.246 mm.	3.232 mm.

23. Problem. — *To find the dimensions of a brass tube and compute the volume of the brass.*

Apparatus. — The **vernier steel caliper** (Fig. 11). If the wooden blocks (21) were attached to the meter rod by clasps so as to slide smoothly along it, the instrument would be essentially a caliper. The vernier caliper is an improvement on this in that it provides for an accurate reading of small parts of the smallest divisions on the long bar. The instrument consists of two parts, a graduated bar called the *limb*, to which one jaw

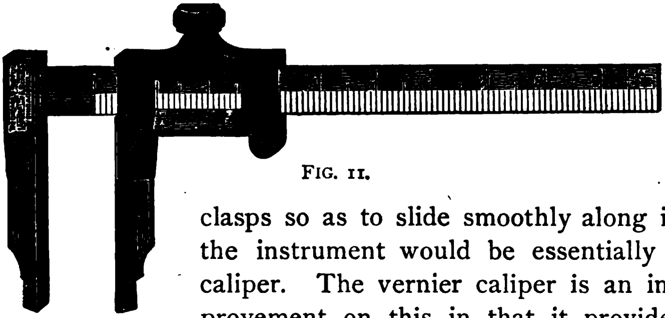

FIG. 11.

is usually firmly fixed, and a sliding jaw carrying an index which is the zero of a scale called the *vernier.* The vernier scale (an enlarged view is given in Fig. 12) is constructed so that n of its divisions equal $n - 1$ divisions of the limb. Hence, each vernier division is $\frac{1}{n}$th of a limb division shorter than each limb division. In Figs. 11 and 12, 9 divisions on the limb equal 10 divisions on the vernier, so that each vernier division is 0.9 of a limb division. To read the instrument, write down decimally the number of whole limb divisions up to the zero line of the vernier scale; then if the zero of the vernier scale does not register exactly with a division on the limb, observe which one of the vernier divisions coincides most nearly with a division on the limb, and this gives the value of the small space between the zero of the vernier and the limb division that is nearest to it on the side toward the jaws. For example, in Fig. 12 the limb reading is 4.6, and the space between the line 0.6 and the zero line (index line) on the vernier is found by running along the line till two coincident lines are found; line 4 on the vernier is the one, hence the space in question is 0.4 of a limb space and the reading is 4.64 units.

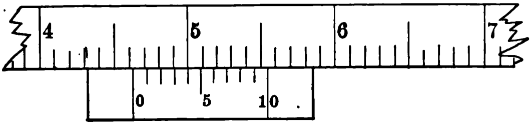

FIG. 12.

Directions. — Unclamp the screw on the back of the caliper, and slide the jaw far enough to admit the cylinder between the jaws, with its axis parallel to the limb. Then move the jaw till the tube is just held between the jaws, and note the reading.

The outside diameter is measured by placing the tube between the jaws with its axis at right angles to their plane. To obtain the inside diameter, insert the rounded ends of the jaws within the cylinder, separate them till contact is secured, and read the vernier. This reading must be increased by the width of the jaws. If this width is not known, it can be found by the aid of a second vernier caliper, or by means of the wire micrometer (22).

The volume required is the difference between the volumes of the two cylinders, whose diameters are the outside and the inside diameters, respectively, of the tube. The volume of a cylinder can be computed by the formula $\pi R^2 H$, in which $\pi = 3.1416$, $R =$ radius, and $H =$ length.

Remarks. — The object must not be clamped too tightly, since it may spring the frame of the instrument, and make the measurements wrong. The screw in the movable jaw should be loosened *just enough* to permit the jaw to slide along; this will reduce to a minimum the amount it springs away from the limb when pressed against the object. Every instrument should be tested at the outset to see what the reading is when the jaws are brought together. This reading should be zero; if not, every reading, as seen in Art. 22, must be corrected by this zero reading.

Form of Record. — The following illustrates the manner of tabulating the measurements: —

	LENGTH	OUTSIDE DIAMETER	INSIDE READING	WIDTH OF JAWS	INSIDE DIAMETER
1st measurement	3.48 cm.	2.25 cm.	1.01 cm.	1.00 cm.	
2d "	3.49 cm.	2.24 cm.	1.00 cm.	1.00 cm.	
3d "	3.47 cm.	2.25 cm.	1.02 cm.	1.00 cm.	
4th "	3.48 cm.	2.24 cm.	1.01 cm.	1.00 cm.	
Mean . . .	3.48 cm.	2.245 cm.	1.01 cm.	1.00 cm.	2.01 cm.

Vol. of brass = 2.733 cm.^{3}.

24. Problem. — *To measure the diameter of a tube, too small for the insertion of the jaws of a vernier caliper.*

Apparatus. — The **tube gauge** (Fig. 13). This gauge is constructed of metal or cardboard, preferably the former. A thin strip is cut to a tapering shape, being made exactly 2 cm. across at a distance of 10 cm. from the point. The edges of the scale must be perfectly straight and beveled to a knife-edge, as shown in section in Fig. 14. Parallel transverse lines are drawn at half-centimeter intervals, and each of these spaces is divided into ten equal parts, the lines of division being confined to the edges. The half-centimeter divisions are numbered 1, 2, 3, 4, etc., the point of the scale being zero. The gauge, as now graduated, will give the bore of a tube in millimeters and tenths of a millimeter.

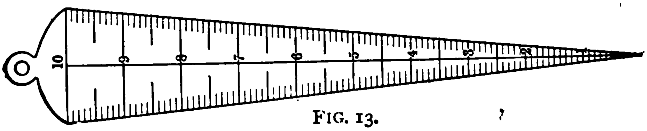

FIG. 13.

FIG. 14.

Directions. — Before measuring the bore of the tube, if metal, file the end to a true plane normal to the axis of the tube; if glass, grind it on a flat piece of fine sandstone. The edges of the tube must be sharp to make it possible to read the gauge with accuracy. Now insert the point of the gauge in the tube till the edges touch the tube diametrically, the axis of the scale and the axis of the tube coinciding. Read the graduation on the scale where the edge of the tube touches it, using a magnifier for the purpose. Measure each end of the tube several times, and take the mean of these several measurements for the diameter of the tube.

Remarks. — The gauge being graduated on both edges makes it easy to determine when the axis of the scale and tube coincide. The method, it will be seen, is a direct one. Under the use of the balance there will be given an indirect method of measuring the bore of small tubes (33).

Form of Record. — The student is to devise a suitable form of tabulation.

25. Problem. — *To measure the volume of a cylindrical jar, both directly and indirectly.*

ht + diam = vol.

Apparatus. — A **meter rod**, a **graduate** (Fig. 15), **inside calipers** (Fig. 16), and a **straightedge**, one edge of which is beveled to a knife-edge. The graduate is a glass vessel of cylindrical or conical form, showing on opposite sides the volume in ounces and cubic centimeters respectively. By observing the number of spaces between the named divisions, the value of each space is easily obtained.

The inside calipers are merely a form of compass. Sometimes they are provided with an adjusting screw, to be used in opening and closing the instrument, as well as in adjusting the distance between the points more accurately.

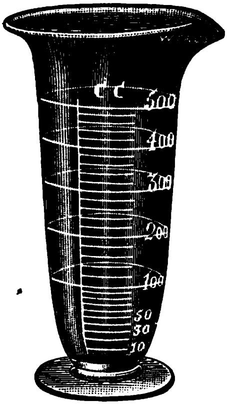

FIG. 15.

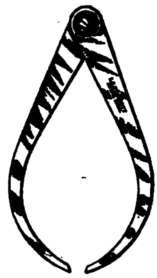

FIG. 16.

Directions. — Find the depth of the jar by standing the meter rod vertically within it, and taking the reading on it at the point opposite the lower surface of a straightedge resting across the top of the jar. Find the inside

dia. ht. by [illegible]

diameter by opening the inside calipers within the jar as far as possible, keeping the line of the points *normal* to the sides of the jar, and afterward finding the distance between their outer faces by applying the points to a linear scale. Compute the volume by the formula $V = \pi R^2 H$, in which $\pi = 3.1416$, $R =$ radius, and $H =$ depth.[1]

To measure the volume with the graduate, fill the graduate, as it rests on a horizontal surface, with water exactly to some known reading, and then empty it into the vessel. Continue in this way till the water in the vessel just touches the lower surface of a straightedge placed across its top. Deduct from the total amount of water poured into the graduate the amount remaining in it after the vessel is full; the remainder will be the capacity of the vessel.

Remarks. — The depth of the jar should be measured at several places, since the bottom may not be level. Likewise the inside diameter should be measured in several places, since the jar may not be a true cylinder. In reading the graduate, the eye must be on a level with the surface of the water. Why? Give three reasons for the computed volume being different from the measured volume.

Form of Record. — The following form may be used in tabulating the data: —

	DEPTH		DIAMETER		MEASURED VOLUME	
	cm.	in.	cm.	in.	$cm.^3$	oz.
First trial . .	——	——	——	——	——	——
Second " . .	——	——	——	——	——	——
etc.	etc.	etc.	etc.	etc.	etc.	etc.
Mean . .	——	——	——	——	——	——

Computed volume = —— $cm.^3$; —— oz.

---

[1] To obtain the volume in ounces, compute the volume in cubic inches and multiply by .5541.

26. Problem. — *To measure the angles of a triangle formed by lines drawn on a page of the notebook.*

Apparatus. — The **circular protractor** (Fig. 17). This instrument, in its simplest form, is a semicircle of translucent horn or celluloid, with its circular edge divided into degrees or half-degrees. Metal may be used instead of horn, the instrument may be a circle instead of a semicircle, and it may be provided with an arm (Fig. 18), turning about the center of the circle, with one of its edges in line with the radius of the circle. This arm may carry a vernier, so that measurements may be made to the fraction of a degree.

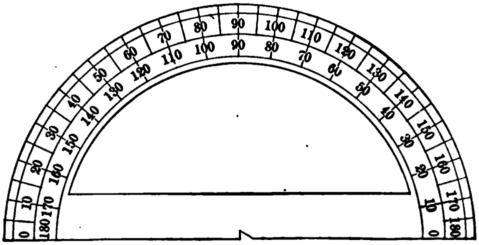

FIG. 17.

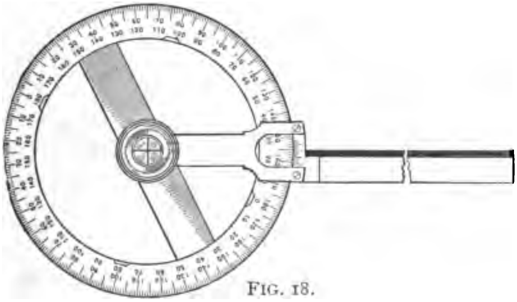

FIG. 18.

Directions. — Place the protractor on the angle, with its center as exactly as possible over the vertex of the angle, and the diameter of the protractor (the line joining the zeroes of the scale) on one side of the angle. Then the point where the other side of the angle crosses the circular scale will mark the size of the angle.

Remarks. — If the sides of the triangle do not extend out to the scale on the protractor, they should be produced till they do. Test the work by the principle that the sum of the angles should be 180°, and compute the per cent. of error in the work.

Form of Record. — The student is to devise a suitable form of tabulation.

27. Problem. — *To find the area of a triangle formed by lines drawn on a page of the notebook.*

Apparatus. — A pair of **dividers** and a **diagonal scale.**

Directions. — Find the length of each side of the triangle by proceeding as in Art. 20. Compute the area by the formula $A = \sqrt{p(p-a)(p-b)(p-c)}$, in which p is the semiperimeter of the triangle, and a, b, and c are the sides, respectively.

Check the result by drawing an altitude to the triangle, measuring this altitude, and computing the area by multiplying the base by half the altitude.

Form of Record. — The following form may be used in tabulating the data: —

	SIDE a	SIDE b	SIDE c	BASE	ALTITUDE
	cm.	cm.	cm.	cm.	cm.
First measurement . . .	——	——	——	——	——
Second " . .	——	——	——	——	——
etc.	etc.	etc.	etc.	etc.	etc.
Mean	——	——	——	——	——

Area by formula = —— cm.^{2}. Check method = —— cm.^{2}.

28. Problem. — *To measure the volume of a small irregular body.*

Apparatus. — A **cylindrical graduate** (Fig. 19), an **Erdmann's float** (Fig. 20), and a liquid in which the solid is not soluble. The graduate is made of glass, and the scale usually reads to cubic centimeters. The float is a short, hollow glass cylinder, closed at both ends, and weighted with mercury to make it maintain an upright position when floating. A line is etched around the middle of the cylinder to serve as an index.

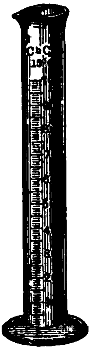

FIG. 19.

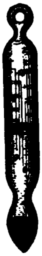

FIG. 20.

Directions. — Partly fill the graduate with a suitable liquid, usually water, introduce the float by means of a wire hook, and record that reading on the scale which is exactly opposite the line on the float. Now remove the float and introduce the object to be measured; then reintroduce the float and obtain the reading as before The difference between the two readings is the volume of the object.

Remarks. — In repeating the work, use different quantities of liquid, in order to bring the readings at different places on the scale. Dry the object each time. In taking a reading, the line of sight should be a diameter of the circular line around the float. Why? Fractions of a cubic centimeter should be estimated in tenths. When the float is lifted out of the graduate in order to introduce the object, some liquid adheres to it, thereby lessening the amount in the graduate. This error may be avoided to a large degree by placing the float, after each removal, in a vessel filled with the liquid, since the probability is that the float will carry as much liquid into the graduate on being returned as it carried out.

It is recommended that the first reading for each determination of the volume be made a whole number, a pipette being used to introduce or remove small quantities of the liquid to make the reading exact.

A substance soluble in water must be measured by using some liquid in which it will not dissolve. Bodies lighter specifically than water must be weighted down by a piece of lead or iron of known volume, the combined volumes obtained, and then corrected by subtracting the volume of the sinker.

All adhering air bubbles must be removed by knocking them off with a wire. Why necessary? Porous substances should have their pores closed with a thin coating of varnish or paraffin. Why necessary? What principle is applied in solving this problem?

Form of Record. — The data of this problem may be tabulated as follows: —

	FIRST FLOAT READING	SECOND FLOAT READING	VOLUME
First measurement . . .	—— cm.^{3}	—— cm.^{3}	——cm.^{3}
Second " . . .	—— cm.^{3}	—— cm.^{3}	——cm.^{3}
etc.	etc.	etc.	etc.
Mean	. . . .	. . . .	——cm.^{3}

II. Measurement of Mass.

29. Measuring the mass of a body consists in finding the ratio of the quantity of matter in it to that in some body taken as a standard. The process is called **weighing**, and the result is commonly spoken of as the **weight**. The unit of mass adopted and in general use in this country and in England is the **avoirdupois pound**. It is equivalent to 7000 grains. Its chief multiples and submultiples are tons, ounces, and drams. In the metric system the **gram** is the unit. For the purposes of this book, it may be considered with sufficient accuracy as the mass of a cubic centimeter of ice water. It is equivalent to 15.432 grains. It is multiplied and divided decimally; the kilogram (1000 grams), the decigram (0.1 gm.), the centigram (0.01 gm.), and the milligram (0.001 gm.) are the denominations most frequently met with in practice.

30. Relation of the Gram to the Pound. — Pounds are reduced to grams by multiplying by 453.593, ounces to grams by multiplying by 28.35, and grams to pounds and ounces by the converse process. For other reduction rules see Appendix, Table XXI.

31. Problem. — *To find the mass of a body.*

Apparatus. — A **beam balance** (Fig. 21) and a **box of weights** (Fig. 22). The balance consists of a metal beam supported on a **V**-shaped axis, about which it is free to vibrate in a vertical plane. Pans of horn or metal are attached by cords or metal bails to the ends of the beam. A long pointer is fastened to the center of the beam, and moves in front of a graduated arc as the beam vibrates. In the better class of balances the beam does not rest on the bearings when not in use, but is set in action either by depressing a lever, or by

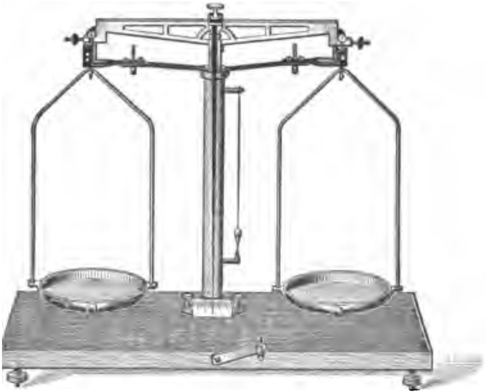

FIG. 21.

FIG. 22.

turning a crank or a milled-head button conveniently placed on the base of the instrument. Fine balances are protected from dust and air currents by a glass case (Fig. 23).

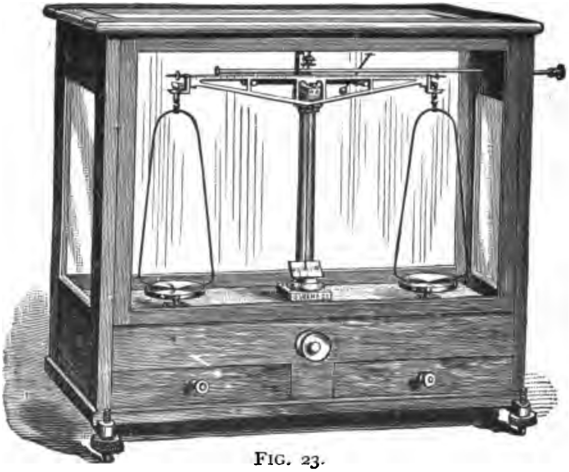

FIG. 23.

The box of weights usually contains weights of the following values in grams: —

50, 20, 20, 10, 5, 2, 2, 1, 0.5, 0.2, 0.2, 0.1, 0.05, 0.02, 0.02, 0.01, 0.005, 0.002, 0.002, 0.001.

In small sets the 50 gm. and often one of the 20 gm. are omitted. The weights of less value than a gram are made either of aluminium or of platinum; the others are made of brass. Each weight has its value stamped upon it, and has its special place in the box. A small pair of pincers is provided for handling the weights; the fingers should not be used for the purpose.

Directions. — Unless the balance has been carefully adjusted to position, the pointer, when the vibrating beam comes to rest, will generally not coincide with the middle division of the ivory scale. It will therefore be necessary to determine the position on the scale that the pointer would have when the beam stops oscillating. To wait for the beam to come to rest in order to find this position or **resting point**, will consume too much time. It can be done more quickly by observing the position of the pointer as it swings back and forth a few times. On account of friction and resistance of the air, the resting point is not the middle point of the arc described by the pointer, but is nearer the end of the arc toward which the pointer moves in making its second vibration. It can, however, be found with sufficient accuracy by observing five turning points of the pointer, and averaging them in the manner shown below. Let us suppose that the ivory scale is divided into twenty parts. Number these divisions from 0 to 20, beginning at the left-hand end (Fig. 24). Now observe five turning points of the vibrating pointer, three to the left and two to the right, or *vice versa*, giving the last swing, the least, to the same side as the first swing, the greatest. To illustrate, let the readings be as follows: —

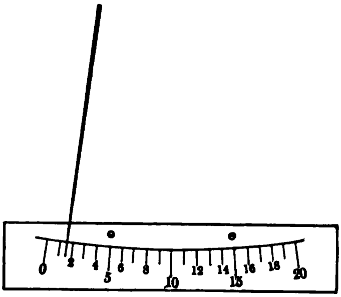

FIG. 24.

TURNING POINTS.

LEFT	RIGHT	RESTING POINT
2.5	17.0	$\frac{3.1 + 16.6}{2} = 9.8$
3.0	16.3	
3.8		
Mean . 3.1	16.6	

The mean of the three vibrations to the left is 3.1, and of the two to the right is 16.6. The average of these two is 9.8, that is, the pointer will stop at that point on the ivory scale when the beam comes to rest, and at this point the pointer should be, whenever the pans of the balance are equally loaded. It should be noticed that the decimal part of these observations is an estimated measurement, and hence is in doubt; therefore, in computing the resting point, there is no gain in accuracy secured by carrying the calculation beyond one decimal place.

As soon as the resting place is found, then place the article to be weighed on the left-hand scale pan, and as near the middle as practicable. In the other pan place a weight such as you estimate will balance it. Now set the balance in action, and watch the pointer to determine which pan is heavier; if the equilibrium is not close, the pointer will move sharply to one side of the middle of the ivory scale. If the weight is found too heavy, remove it, and substitute a smaller one; if too small, a heavier one. Then, without missing any, add each smaller weight successively, removing it if found too heavy, leaving it if too light. Then, when sufficient weights have been added to make the pointer swing past the determined resting point, record five of its turning points and compute the resting point as before. To illustrate, let us suppose that this resting point is 7.2, the true one being 9.8. Then it appears that there are too many weights in the right-hand pan, since the resting point is to the left of the true one, the one for the unloaded balance. To determine the correction, or the amount to be subtracted, place a small weight, as 0.001 gm. on the pan, set the balance in action, and determine as before the resting point. To illustrate, let us assume that the result obtained is 6.4. Then it follows that 0.001 gm. moved the pointer from 7.2 to 6.4 or 0.8 division. To secure an exact balance it is necessary, however, to move the pointer from 7.2 to 9.8, or 2.6 divisions. Therefore, the amount required will be given by the proportion,

$$0.8 : 2.6 :: 0.001 : x, \text{ from which } x = 0.0032 \text{ gm.}$$

This amount, deducted from the weight on the pan, not counting the milligram weight last added, will be the weight sought. When the second resting point exceeds the first, the computed correction must be added. Counting the weights on the pan should be done by writing down in the notebook the weights absent from the box, and then checking the result as you remove them from the pan to replace them in the box.

The milligram weights are so small that they are liable to be lost, and, moreover, they are not necessary for close weighing with the finer balances. The arms of such a balance are divided into ten equal parts, and by means of a brass rod extending through the balance case and parallel to the beam (Fig. 23) a small piece of bent wire called a **rider**, and weighing usually one centigram, can be placed astride of the beam. If the rider is placed on the first division from the fulcrum, it becomes equal to a milligram weight placed in the pan; if placed on the second division it equals a two-milligram weight, and so on.

Remarks. — The following summary of directions must be carefully observed if accurate results are expected: —

1. Remove all dust from the pans by means of a camel's-hair brush.

2. The position of the observer *must be central*, so that the line of sight will be normal to the ivory scale and thus *avoid parallax* (19) in taking the readings.

3. If the balance fails to vibrate on turning the milled head, throw air against one of the pans by a sudden downward motion of the open hand.

4. *In arresting the balance, always wait till the pointer is over the center of the scale.* To do otherwise will jar the instrument, and possibly injure the bearings.

5. *Always arrest the balance before adding or removing weights.* If there is no provision for the purpose, steady the pan with the left hand.

6. *Keep the weights in the box when they are not in use on the scale pan.* Ascertain the number of weights in the box before beginning to weigh, and be sure that *each weight is in place when the operation of weighing is completed.*

7. Place the box of weights near to the right-hand pan, to reduce the liability of dropping weights on the floor.

8. The balance should be placed on the table so that there is room for the notebook between the balance and the edge of the table.

Form of Record. — Tabulate as follows: —

UNLOADED BALANCE		LOADED BALANCE		0.001 GM. ADDED TO PAN		WEIGHTS ON PAN
2.0	17.5	2.5	18.0	2.5	16.5	10
2.5	17.0	3.0	17.5	3.0	16.0	5
3.0		3.5		3.5		2
2.5	17.2	3.0	17.7	3.0	16.2	1
						.05
R.P. = 9.8		R.P. = 10.4		R.P. = 9.6		.01
						.005
Correction to be added, $0.8 : 0.6 :: 0.001 : x$. $\therefore x =$ 0.0008 gm. True weight, $18.065 + 0.0008 = 18.0658$ gm.						18.065 gm.

32. Alternative Method. Apparatus. — **A Jolly balance** (Fig. 25), and a **box of weights**. The illustration shows the balance in one of its latest patterns. In the Appendix (135) directions are given for constructing a simple form that will be found quite efficient. The essential part of the instrument is a delicate spiral spring suspended in front of a linear scale, usually one cut on mirror plate. A scale pan for holding the body to be weighed is attached to the lower end of the spring, and a small bead just above the pan serves as an index. The scale reading is where the line joining the index with its reflection intersects the scale. The instrument is an application of Hooke's law, that is, in elastic bodies the strain is proportional to the stress.

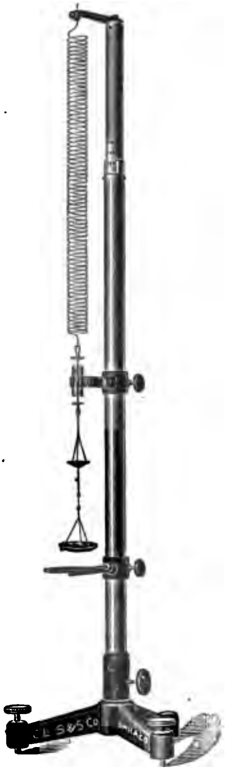

FIG. 25.

Directions. — Place the body on the pan and record the reading of the index. Now remove the body, and add weights till the same reading is obtained. The sum of the weights is the weight of the body. If it is found impossible to bring the index exactly to the same reading as it had when the body was in the pan, add weight till a reading less than the required one is obtained, but such that the centigram weight brings the index too low, and then proceed as in the following example. Let 300 be the required reading, 299 the nearest reading that can be obtained less than 300, and 301.5 the reading when 0.01 gm. is added. Then the true weight will be the weights in the pan, exclusive of 0.01 gm., increased by $\frac{1}{301.5 - 299}$ or $\frac{1}{2.5}$ of .01 gm. = .004 gm.

Remarks. — The spiral spring used should be such that a centigram weight moves the index through two or more divisions of the scale.

Form of Record. — The following example illustrates the manner of recording the data : —

		Weights on Pan
First index reading	300	10
Second "	299	2
Third "	301.5	.5
		.2
		12.7 gm.

Correction . . . . $\frac{1}{301.5 - 299} = 0.004$ gm.

True weight . . . . $12.7 + 0.004 = 12.704$ gm.

33. Problem. — *To measure the diameter of a very small glass tube.*

Apparatus. — A **balance, box of weights, linear scale, dividers, mercury, dropper-bulb**, a short piece of small rubber tubing, hydrochloric acid, alcohol, and distilled water.

Directions. — Thoroughly clean the tube by washing it, first in hydrochloric acid, secondly in distilled water, and lastly in alcohol. To do this, attach a small dropper-bulb to a piece of glass tubing, connect this glass tube by a piece of rubber tubing to the tube to be cleaned; then place one end of the tube in a short bottle containing the cleaning liquid, and by alternately compressing and relaxing the bulb the washing liquid will be made to pass to and fro through the tube. Dry the tube by passing a stream of hot air through it, or by holding it in the heated gases rising above a Bunsen flame. Partially fill the tube with pure mercury by submerging it in a dish of mercury. While submerged, close the ends with the thumb and finger; then lift the tube out, brush off all clinging globules of mercury with a soft brush, tilt the tube till the mercury filament moves to the middle portion of the tube, and then lay it down on a level surface while measuring the length of the filament. This can be done indirectly as in Art. 20, or directly by placing the tube on a finely divided steel scale, and reading the position of the ends of the mercury column by the aid of a magnifier. Now empty the mercury into a small glass vessel whose weight has been determined and find its weight. Since a cubic centimeter of mercury at 20° C. weighs 13.55 grams, then the weight of the mercury in grams divided by 13.55 will give the volume of the mercury cylinder in cubic centimeters. This volume divided by the length of the cylinder will be the area of the cross-section of the tube. Why? By dividing this cross-section by 0.7854 and extracting the square root of the quotient, the diameter of the tube is obtained. Why? Compare the result with that obtained by means of the tube gauge (24).

Remarks. — Since the mercury does not fill the tube completely on account of a thin film of air between it and the walls of the tube, the computed diameter should be less than the true diameter. This will be offset by measuring as the length of the mercury cylinder the distance between the points where the curvature of the ends begins, the small amount of mercury comprised in the meniscus being assumed to be approximately equal to the volume of the air film.

Form of Record. — The data may be tabulated after the following plan: —

Weight of the Empty Glass Vessel.

Unloaded Balance		Loaded Balance		0.001 gm. added to Pan	
2.0	17.5	2.5	18.0	2.5	16.5
2.5	17.0	3.0	17.5	3.0	16.0
3.0		3.5		3.5	
Resting point = 9.9		Resting point = 10.4		Resting point = 9.6	

Weights on Pan	Length of Mercury	Correction to be Added
10	5.12 cm.	$0.8 : 0.5 :: 0.001 : x$ $\therefore x = 0.0006$ gm.
5	5.13 "	
2	5.11 "	
1	5.12 "	
.05	5.11 "	True Weight of Vessel
.01	5.13 "	
.005		
18.065	5.12 cm.	18.065 + 0.0006 = 18.0656 gm.

Vessel and Mercury.

Unloaded Balance		Loaded Balance		0.001 gm. added to Pan	
2.0	17.5	1.5	17.1	2.3	15.0
2.5	17.0	2.0	16.9	2.8	14.4
3.0		2.5		3.9	
Resting point = 9.9		Resting point = 9.4		Resting point = 8.8	

VESSEL AND MERCURY — *Continued.*

WEIGHTS ON PAN	CORRECTION TO BE SUBTRACTED, $0.6 : 0.5 :: 0.001 : x, \therefore x = 0.0008$ gm.
20	True weight of vessel and mercury = $43.53 - 0.0008 = 43.5292$ gm.
10	Weight of mercury = $43.5292 - 18.0656 = 25.4636$ gm.
10	Volume of mercury = $25.4636 \div 13.55 = 1.8792$ cm.^{3}
2	Area of cross-section = $1.8790 \div 5.12 = 0.3670$ cm.^{2}
1	Diam. of tube = $\sqrt{0.3670 \div 0.7854} = 0.68$ cm.
.5	Diam. by tube gauge ≐ 0.67 cm.
.02	
.01	
43.53 gm.	

34. For further drill in the use of the balance the following problems are proposed, with suggestions regarding their solution: —

a. Given two pieces of wire of the same material and diameter, the one straight and the other crooked, to find the length of the crooked piece.

SUGGESTION.—Weigh each piece (31). Measure the length of the straight piece (23). From these data compute by proportion the length of the crooked piece.

b. Given a quantity of shot of uniform size to find the number in a pound.

SUGGESTION.—Find the mass of a known number (31) and from this compute by proportion the number in a pound.

c. Given an ounce weight, avoirdupois; find by the balance the number of grams in it. Reduce the ounce to grams and compare.

d. Find by weighing each how many 10-cent pieces can be made out of a silver dollar.

CHAPTER II.

TENACITY AND ELASTICITY OF SOLIDS.

35. Problem. — *To measure the tenacity of a wire.*

Apparatus. — A **draw scale** or a **spring balance** (Fig. 26) of 15 kgm. capacity, a **wire micrometer** (22), a **tin pail** of about 5 l. capacity, 15 kgm. of lead or iron weights, and several meters of the wire to be tested. The pail should have a wooden handle fitted to the bail. Shot or nails may be used for weights.

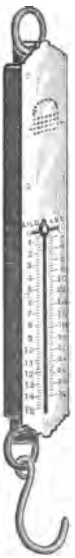

FIG. 26.

Directions. — Suspend the pail by the wire to be tested from a stout iron rod clamped so as to project over the edge of the table. There must be no kinks or short bends in the wire. Hence, in attaching it to the rod, it must be wound around it a number of times to prevent slipping, and the end attached to the clamp. In attaching the wire to the pail, wind it several times about the wooden cylinder and then fasten the end. Use a length of wire such that the bottom of the pail will come within a few inches of the floor. Introduce enough weight in the pail to straighten the wire, and then measure the diameter of the wire in several places (22). Now add weights to the pail, large ones at first and small ones as the breaking point is approached, till the wire breaks. Weigh the pail and contents with the spring balance. Repeat the experiment, using a new piece of wire each time. The arithmetical mean of the several trials may be considered as the breaking weight. Compute the cross sectional area by the formula πR^2, the diameter being taken as the mean of all the measurements. The tenacity will equal the quotient of the mean breaking weight by the cross-sectional area.

Remarks. — A pan of sawdust or sand placed beneath the pail will deaden the noise made by the pail on striking the floor. The distance of the pail from the floor must be greater than the elongation of the wire before breaking.

Form of Record. — The following example illustrates the manner of tabulation: —

KIND OF WIRE	DIAMETER	CROSS-SECTION	BREAKING STRAIN	TENACITY
Iron	0.286 mm.		3.25 kgm.	
"	0.285 "		3.10 "	
"	0.287 "		3.45 "	
Mean .	0.286 "	0.06424 mm.^{2}	3.27 "	50.9 kgm.

36. Problem. — *To calibrate a spiral spring and find its modulus.*

Apparatus. — A **spiral spring**, *B* (Fig. 27), suspended from a suitable support, *A*, in front of a half-millimeter scale drawn on a strip of mirror plate (132) or a wooden scale, *C*, similarly graduated, and a **box of metric weights.** The spring may be made by winding spring brass wire, No. 20, 22, or 24, B. and S. gauge, closely on an iron rod 1 cm. in diameter. A light scale pan, *E*, is attached to the lower end of the spring, and an index, *D*, is added just below the spring. The Jolly balance (32) makes an excellent substitute for the above.

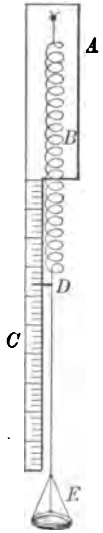

FIG. 27.

Directions. — Remove the scale pan and then record the reading of the index. Weigh the scale pan, attach it to the spring, and then add enough weights to make the load 5 gm., and record the reading of the index. Remove the load and record the reading of the index. This should be done after each load. Increase the load by successive additions of 5 gm. and record the reading of the index. Continue in this way till the limit of the scale is reached. Compute the average reading without load, taken before and after the load was applied. Compute the elongation for each load. Compute the mean elongation per gram and the load that produces an elongation of 1 cm. The force that produces a unit elongation is called the *modulus* of the spring. Compute a table giving the value in grams of each centimeter division of the scale. Plot a curve (15), using the successive loads and the corresponding elongations as the coördinates. Such a curve is known as a *calibration curve*, since from it any elongation of the spring can be interpreted in grams.

Remarks.—In obtaining the average reading without load it should be noticed that the reading taken after removing the load will be the reading before adding the next load. What does it signify if the elongation per gram is constant? If the load be considered as a *stress* and the elongation as a *strain*, what relation between stress and strain is true of an elastic body? When does the law fail? Illustrate in the discussion the use of the calibration curve.

The stretching of a spiral spring is quite a complex phenomenon, since the wire is both twisted (38) and bent (37). Both theory and experiment show that the extension is directly proportional to the pulling force, to the length of the wire, to the diameter of the coils, and inversely proportional to the fourth power of the diameter of the wire.

Form of Record. — Tabulate the data as follows: —

No. of Spring ———

Reading without Load	Average without Load	Reading with Load	Load	Elongation	Elongation per gm.
——	——	——	——	——	——
——	——	——	——	——	——
——	——	——	——	——	——
——	——	——	——	——	——
etc.	etc.	etc.	etc.	etc.	etc.

Modulus = —— Mean ——

37. Problem. — *To verify the laws of elasticity of bending as exhibited by uniform bars.*

Apparatus. — Three wooden bars, as nearly alike as possible in material, each 1.1 m. long, the first 1.5 cm. wide and 0.5 cm. thick, the second 3 cm. wide and 0.5 cm. thick, the third 3 cm. wide and 1 cm. thick. Figure 28 shows a special device for making the test. *A* and *B* are pyramidal

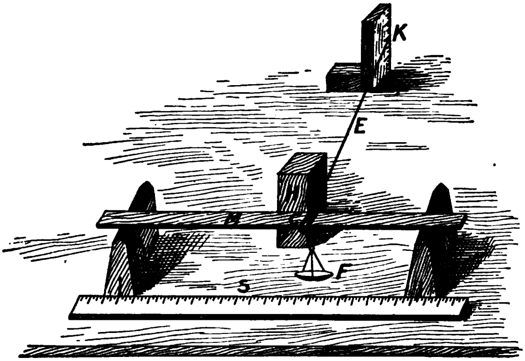

Fig. 28.

pieces of wood to support the bars. *S* is a scale to measure the distance between the supports. *C* is a clevis resting on the bar to which a scale pan is attached. *E* is a lever to measure the deflection on the vertical scale *K*. The parts *A*, *B*, *H*, and *D* rest on the experimental table.

Directions. — Place the supports, *A* and *B*, at 1 m. apart. Support the smallest bar in position with the clevis, *C*, exactly midway between the supports. Record the reading of the pointer. Add weights to the pan till clevis, pan, and weights together make, say, 50 gm., and record the reading of the pointer. Remove the load and again record the reading of the pointer. This should be done after each load. Increase the load by successive additions of 50 gm. and record the reading of the index. Continue in this way for several loads, stopping when permanent bending is produced. Compute the average reading without load taken before and after load was applied. Compute the change of the index reading for each load, and also per 100 gm.. The change of the index reading, being proportional to the flexure, may be considered as the flexure. Plot a curve, using the change in index readings and the loads as coördinates (15). How are flexure and load related according to the data?

Substitute for the first bar the one which is twice as wide. Keep the distance between the supports exactly as in the first case and repeat the work, using the same loads. Reduce the data as before and plot a curve. Compare the mean flexure per 100 gm. with that of the first case, and state as a law the effect of width on elasticity of flexure.

Now place the third bar in position, and repeat all the tests of the preceding cases. Reduce the results and plot a curve as before. Multiply the mean flexure per 100 gm. by the cube of the thickness of the bar in both the second and third case and compare the results. What effect does thickness seem to have on elasticity of flexure?

Place the first bar again on the supports, but make the distance between these supports 50 cm. Repeat all the tests as before. Reduce the data and plot a curve. Compare these results with those of the first case by dividing the mean flexure per 100 gm. in each case by the cube of the length or distance between the supports. What effect does length seem to have on elasticity of flexure?

Remarks. — Variations in the wood will be largely responsible for the deviations of the results from the law. Metal bars would give more concordant results. As in the preceding problem, it should be noticed that, in obtaining the average reading without load, the reading taken after removing the load will be the reading before adding the next load. It is not necessary that the first load should be 50 gm., or that the change each time should be by that amount. This must be determined by the stiffness of the bars.

Form of Record. — The results may be tabulated as follows: —

Case I. — Bar made of ——, length, —— breadth, ——, thickness, ——, distance between supports, ——.

READING WITHOUT LOAD	AVERAGE WITHOUT LOAD	READING WITH LOAD	LOAD	FLEXURE	FLEXURE PER 100 GM.
——	——	——	——	——	——
——	——	——	——	——	——
——	——	——	——	——	——
——	——	——	——	——	——
etc.	etc.	etc.	etc.	etc.	etc.

Mean . . . ——

38. Problem. — *To verify the laws of elasticity of torsion as exhibited by uniform rods.*

Apparatus. — Figure 29 illustrates an easily constructed device for this problem. It consists of a board, *B*, with an upright piece, *A*, rigidly braced, supporting a pointer, *D*. The rod, *F*, to be tested is secured firmly in a small vise, *H*, clamped to the board; the other end of the rod is supported by the upright, *A*. A wheel, *C*, is fitted with a metal hub and set screw, through which the rod passes, and by which the wheel is clamped to it. A scale pan, *E*, is attached to the perimeter of the wheel by a cord.

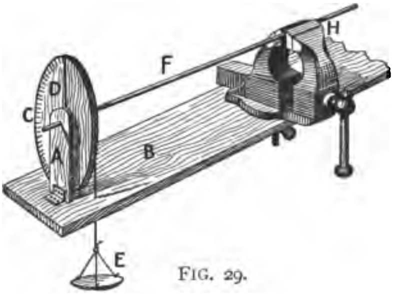

FIG. 29.

Directions. — Adjust the rod, *F*, so that the vise, *H*, is applied near its center, and measure very carefully the distance between the hub and the vise. Measure the diameter of the rod in several places (22). Adjust the rod so the index reads zero. Place, say, 50 gm. in the scale pan and record the reading of *D*. Remove the load and again note the reading. This should be done after each change of load. Now place 100 gm. in the scale pan and record the reading. Proceed in this way for several changes of load.

Repeat the work with the vise at the end of the rod. Also repeat the work for a rod of larger diameter and of the same material, adjusting the rod to the same lengths as in the first case.

Compute the average of the two index readings, the one before applying the twisting force, and the one after the twisting force is removed. Compute the change of index reading for each load. Plot a curve for each case, using the change in index readings and the loads producing these changes as coördinates. Divide each change in index reading by the corresponding load or twisting force. What law will express the relation between the twisting force and the amount of twist? Divide the deflection for each twisting force by the length of the rod. What law will express the relation between the deflection and the length? Multiply the deflection by the fourth power of the diameter. What law will express the relation between the deflection and the diameter of the rod?

Remarks. — The results used in drawing conclusions should be the arithmetical mean of several trials, the conditions remaining unchanged. In comparing results, to ascertain whether any changes can be expressed as laws, it must be borne in mind that when the effect of length is in question, the diameter must be constant, and conversely.

Form of Record. — The data may be tabulated in a manner similar to that employed in the preceding problem.

CHAPTER III.

MECHANICS OF SOLIDS.

I. Composition of Forces.

39. Problem. — *To verify the rule for compounding concurring forces acting at an angle.*

Apparatus. — Three **draw scales,** a sheet of paper, three **iron clamps,** a **circular paper protractor** reading from 0° to 360°, a **horn protractor,** and a piece of stout twine. Each draw scale should be fitted in a cradle made by tacking two wooden strips to a narrow thin board, forming a trough, in which the balance rests face upward. This keeps the hook of the balance from dragging on the table, and also furnishes means for clamping the balance to the table. The ring of the balance is fastened to the cradle by a screw.

Directions. — Tie together at one end three pieces of the twine, each piece being about 30 cm. long, and fasten the free ends to the hooks of the balances respectively. Clamp two of the balances some distance apart on one side of the table and the third balance on the opposite side, regulating their relative positions and the length of the cords so that each balance registers several divisions of the scale. Carefully adjust each cradle so that the hook bar of each balance does not rub against the balance frame. There are three forces, then, in equilibrium acting at *D* (Fig. 30), any one of these three being equal and opposite to the resultant of the other two. Place beneath the cords a thin board on which the paper protractor is tacked. Bring the center of the protractor exactly beneath the juncture of the cords, and at the same time have one cord exactly over the zero line of the protractor. Now read the position of each cord and compute the angles between them. Also read the forces as measured by the draw scales. Now, on a sheet of paper, by the aid of the horn protractor, draw three lines representing the cords. Then proceed in the usual way to lay off on each line as many units of length as there are units of force indicated by the respective balance. Complete a parallelogram, using two of these measured lines as adjacent sides. Draw the diagonal that passes through the common point of application of the forces, and measure its length by the scale adopted in the drawing. This will give the resultant of the two forces, and it should agree in value with the third

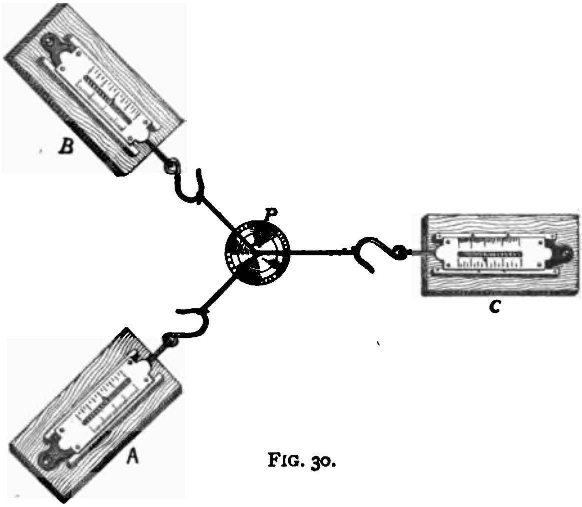

FIG. 30.

force. Compute the error. Make several trials, varying the angles between the directions of the forces, as well as the magnitudes of these forces.

Remarks. — By using additional balances, the principle known as the *polygon of forces* can be verified. The paper containing the completed figures should be inserted in the permanent notebook, or the figures should be carefully transferred to the pages of that notebook. The figures should be lettered to fit the tabulated data, and the scale to which each figure is drawn should be marked by the side of the figure. The draw scales should be compared, each reading being corrected for any error in the zero.

Form of Record. — Tabulate the results as follows: —

	Balance *A*		Balance *B*		Balance *C*		Angle Readings of the Cords			Angles between Forces		Computed Resultant of *A* and *B*	Error
Zero reading	——		——		——		*A*	*B*	*C*	*A* and *B*	*A* and *C*		
	Uncorrected	Corrected	Uncorrected	Corrected	Uncorrected	Corrected							
First trial	—	—	—	—	—	—	—	—	—	—	—	—	—
Second "	—	—	—	—	—	—	—	—	—	—	—	—	—
etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.

40. Problem. — *To verify the rule for compounding parallel forces acting in the same direction.*

Apparatus. — Two **draw scales**, a light **graduated wooden bar** 0.5 m. long, a **weight** of known value, say, 200 gm., a rod supported horizontally over the table, and a lead ball and thread for a plumb line. If a meter rod be cut in two lengthwise, each half will make two graduated bars.

Directions. — Suspend the draw scales from the horizontal rod, with the graduated bar hanging from the hooks by means of wire or thread stirrups. Adjust the draw scales to vertical positions by the use of a plumb line. Read each draw scale to find the effect of the graduated bar. Now attach the weight at some point along the bar (Fig. 31), and readjust the draw scales to vertical positions. Read the draw scales and also the distances from the point of attachment of each on the bar to that of the weight. Find the effect of the weight on each draw scale, that is, find the parallel forces involved. Compare the sum of these forces with the weight. Compute the ratio of these forces, and compare it with the inverse ratio of the distances of their points of application from that of the weight. Make several trials, using different positions for the weight.

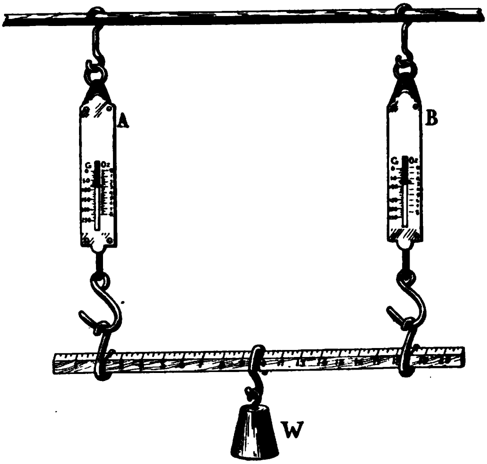

FIG. 31.

Remarks. — The balances should be compared, that is, one calibrated by the other, by hooking one into the other and drawing them out, noting the readings of their indexes at a number of points. If their readings differ, suitable corrections must be applied to the readings obtained in solving the problem. If the pointer does not start on zero, a correction will be necessary. Why must the balances be parallel?

Form of Record.—The following example illustrates a convenient form of tabulation:[1]—

	BALANCE *A*	BALANCE *B*	WEIGHT *W*	RESULTANT $A+B$	ERROR
	gm.	gm.	gm.	gm.	gm.
First reading	15	18			
Final "	120	89			
True "	105	71	180	176	4
First reading	15	18			
Final "	112	98			
True "	97	80	180	177	3
etc.	etc.	etc.	etc.	etc.	etc.

	POSITION OF *A*	POSITION OF *W*	POSITION OF *B*	DISTANCE *A* TO *W*	DISTANCE *B* TO *W*	RATIO OF FORCES	INVERSE RATIO OF DISTANCES	ERROR
	cm.	cm.	cm.	cm.	cm.			
First r'd'g								
Final "								
True "	8.0	23.7	48.0	15.7	24.3	1.48	1.55	– 0.07
First r'd'g								
Final "								
True "	8.0	26.0	48.0	18.0	22.0	1.21	1.22	– 0.01
etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.

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[1] In the class notebook the second part of this tabulation should appear to the right hand of the first part and not below it as here shown, unless the page is narrow.

II. Curvilinear Motion.

41. Problem. — *To verify the laws of curvilinear motion.*

Apparatus. — An **iron ball** weighing two or three kilograms, a **draw scale** reading to grams, a **clock** ticking seconds, a brass wire stout enough to support the ball, and a **meter rod.**

Directions. — If the weight, W (Fig. 32), be suspended from A by a wire, AW, and AW be made to describe a cone about the vertical AB, W describing a circle about B, then W is maintained in its path by a centrifugal force which is equal to that required to maintain the weight at W if applied to it in the direction of BF. Hence, if the centrifugal force of W be computed by the formula $F = \frac{Mv^2}{gr}$ in which M is the mass of the ball in grams, v is its velocity in cm. per sec. in its circular path about B, of which r is the radius in cm., and g is the acceleration due to gravity, or 980 cm. for the latitude of New York, it should equal the force in grams necessary to pull the weight to a position, W, distant r from B. This force may be measured by applying a draw scale.

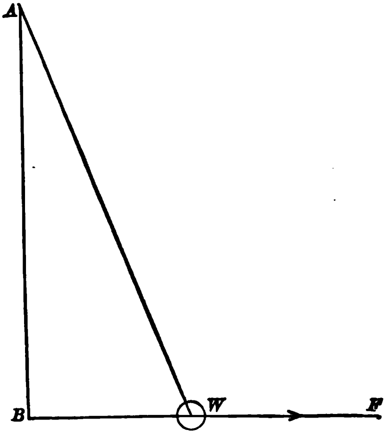

FIG. 32.

On account of the difficulty of making the ball swing in a circle, and the additional difficulty of determining the radius of the path when so swinging, it is better to make the ball swing as an ordinary pendulum, the chord of the arc of vibration to be the diameter of the circular path, and determine the time of a swing to and fro; that is, the time of a complete vibration. This time will be approximately the same as that required to complete the orbit whose diameter is the chord of the arc.[1]

Place the meter rod exactly beneath the pendulum and determine the force by inserting the hook of the draw scale in a loop about the ball, necessary to hold the ball, say, at a point 30 cm. from the vertical. The draw scale must be kept exactly horizontal in measuring this force. Then start the ball swinging to and fro from this point, and find the time of a single vibration. To do this, proceed as described in Art. 43. The time of a complete vibration and the radius of the circle will be sufficient data to determine the velocity of the conical pendulum in its circular orbit. Now, by the formula, compute the centrifugal force and compare with the force as given by the draw scale. Compute the difference.

Remarks. — In pulling the ball to one side by means of the draw scale in order to measure the centrifugal force, the scale should be attached to a point on the ball opposite the center of mass. This can be readily done by tying a string around the ball. The measurement of radius should be made to the center of the ball. As already stated, the time of a complete vibration is approximately that required to describe the circular orbit. The correct time can be obtained by multiplying this observed time by $\sqrt{\frac{l}{l'}}$, in which l' is the distance from the point of suspension to the center of the ball, or AW, and l is the vertical distance between the point of suspension and the horizontal plane through the center of the ball when at the extreme point of its orbit, or AB. By using balls of different sizes the effect of mass can be shown, and by changing the radius, the combined effect of radius and velocity can be brought out.

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[1] To be exact, the time required to complete a revolution is that required to make a complete vibration by an ordinary pendulum whose length is the projection, *AB*, of the conical pendulum, *AW*, on the vertical, *AB*.

Form of Record. — The results may be recorded as follows: —

M. Kgm.	r Cm.	Vibrations in Seconds		t Single Vib.	v Cm.	Observed C. F. in Gm.	Computed C. F. in Gm.	Difference. Gm.
		No. Vib.	No. Sec.					
2.25	30	26	34	1.308		390		
2.24		33	43	1.303		400		
2.25		29	38	1.310		400		
2.23		39	51	1.308		400		
2.25		29	38	1.310		395		
2.244	30			1.308	72.06	397	396	1

III. Accelerated Motion.

42. Problem. — *To verify the laws of accelerated motion.*

Apparatus. — The **Atwoods machine** as improved by Mr. W. H. Hawkes (Fig. 33). This machine consists essentially of a pendulum, a marking device controlled by it, and an aluminium wheel mounted upon cone bearings and carrying upon its rim a long beltlike loop of tissue paper ribbon, upon which the record is to be made. Two equal weights are attached to this ribbon, one on each side of the wheel, and a small overweight is placed on one side to cause that side to descend. The pendulum is adjusted by the leveling screws in the base of the machine, so that when not swinging, the needle point at its lower end rests in the center of a mercury globule. The marking device at the highest point of the wheel is a very fine camel's-hair brush carrying ink. It is attached to the armature of an electro-magnet in electrical circuit with the pendulum, and hence is actuated at each vibration of the pendulum. There is also connected to this armature a "dog," which keeps the wheel from revolving when the battery circuit is not closed, and releases the wheel at the first movement of the armature. On the underside of the wheel there is an electric brake for stopping its motion at any desired point. A three-point switch on the supporting column places all these moving parts under the easy control of the operator. The paper band being a complete band, introduces no error into the moving system by changing the relative value of the weights attached, since the amount of ribbon on each side of the wheel remains constant.

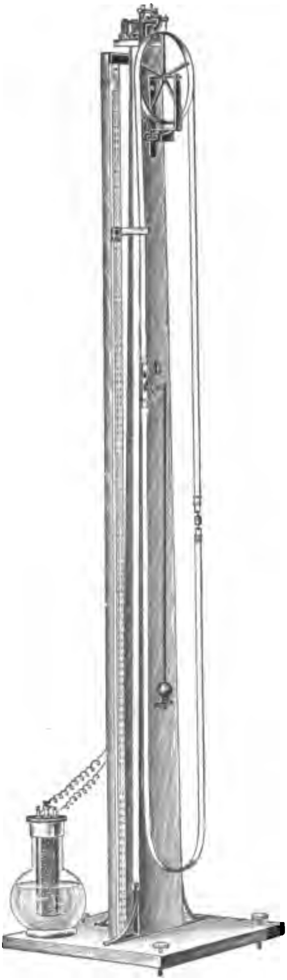

FIG. 33.

Directions. — Put the machine in adjustment with the weight carrying the overweight at the highest possible point. Set the pendulum swinging through a small arc. Now close the pendulum circuit by means of the switch. At the first passage of the pendulum point through the mercury globule, the wheel is released and the first mark is made upon the ribbon. The overweight will cause the wheel to revolve, the overweight descending with a uniformly accelerated motion. As the motion continues, the spaces passed over by the moving system of weights for the successive equal intervals of time as given by the pendulum are recorded by the marker. When the descending weight has nearly reached its lowest point, apply the electric brake by turning the switch to the third point.

Detach the ribbon from the machine, and with a meter rod measure carefully the distance of the upper edge of each mark from the first one. Measure these distances at least five times, record, and find the mean.

From the data compute the distance passed over during each unit of time. Compute the velocity at the end of each unit of time. Compute the acceleration for each unit of time. Compute the ratio of the total distance passed over at the end of each unit of time to that traversed the first one. Represent the distance traversed the first unit by $\frac{a}{2}$, and then show what formula will represent the total distance traversed during t units.

Find the ratio of the distance passed over each unit of time to that passed over during the first. What formula will represent the distance passed over in the tth unit?

Find the ratio of the velocity at the end of each unit of time to that at the end of the first. What relation does the velocity at the end of the first unit bear to the distance passed over during that unit? What formula expresses the velocity at the end of the tth unit?

Is the acceleration constant? Why should it be? Compute the average acceleration. With this mean acceleration compute the total distance the system should move during the successive intervals of time. What is the difference between each observation and the computed distance?

Using the time and total distances traversed as coördinates, represent the data by a curve (15). What kind of a curve is it? Compare its form with the path of a projectile.

Form of Record. — The following record shows the capabilities of the machine, and how to tabulate the work and make the calculations: —

1	2	3	4	5	6	7
cm.	cm.	cm.	cm.	cm.	cm.	cm.
2.31	9.32	20.95	37.11	57.98	83.35	113.38
2.31	9.30	20.91	37.13	57.93	83.37	113.39
2.30	9.30	20.89	37.11	57.92	83.35	113.37
2.32	9.31	20.90	37.12	57.93	83.36	113.35
2.33	9.29	20.91	37.10	57.94	83.35	113.38
2.31	9.30	20.91	37.11	57.94	83.36	113.37

TIME *t*	TOTAL DIST. *S*	DIST. EACH UNIT *S'*	VELOCITY *v*	ACCELERATION *a*	*S* RATIOS	*S'* RATIOS	*v* RATIOS	*a* RATIOS	COMPUTED *S*	DIFFERENCE
	cm.	cm.	cm.	cm					cm.	cm.
1	2.31	2.31	4.68	4.68	1.00	1.00	1.00	1.00	2.31	0.00
2	9.30	6.99	9.30	4.62	4.03	3.01	1.99	0.98	9.24	0.06
3	20.91	11.61	13.89	4.59	9.05	5.03	2.97	0.98	20.79	0.12
4	37.11	16.20	18.52	4.63	16.07	7.01	3.96	0.99	36.96	0.15
5	57.94	20.83	23.11	4.59	25.08	9.02	4.94	0.98	57.75	0.19
6	83.36	25.42	27.70	4.59	36.09	11.00	5.92	0.98	83.16	0.20
7	113.37	30.01			49.08	12.99			113.19	0.18
			Mean	4.62						

EXPLANATION. — *S'* is obtained by subtracting each value of *S* from the succeeding one; *v* is obtained by subtracting the first value of *S'* from the succeeding ones; *a*, the difference between successive values of *v*.

IV. The Pendulum.

43. Problem. — *To verify (a) the laws of the pendulum and (b) to find the value of g.*

Apparatus. — A **lead or iron ball** is suspended by a wire, No. 28, B. and S. gauge, from a frame clamped to the table. The support of the pendulum is a clamp made by fastening a rectangular piece of wood to a second piece by means of screws, and pinching the wire between them. A **meter rod, clock** marking seconds, and a **pair of calipers** complete the outfit.

Directions. — Set the pendulum swinging through a small arc, about 15 cm. long for a pendulum 1 m. long, and find the time of a single vibration. To do this with a fair degree of accuracy two persons are necessary, one to count the vibrations of the pendulum and the other the ticks of the clock. The former should stand directly in front of the pendulum while counting its vibrations, and since the count must begin when the pendulum ball passes through the lowest point of its arc, there should be a mark on the pendulum frame to indicate this position. The observer's eye should not follow the ball through its swing, otherwise he cannot tell accurately when the pendulum passes the line. A good plan is to support a paper tube about 30 cm. long and 2 cm. in diameter with a thread cemented across one end as a diameter. Set the tube so that the thread, pendulum wire, and the mark on the frame are in line. Then as the pendulum swings, the count is made when the pendulum wire crosses the line of sight. When the pendulum crosses the line of sight coincident with the tick of the clock, the observer announces the fact by saying "zero" aloud. He then goes on counting to himself the successive passages of the pendulum, while the other observer counts the ticks of the clock. When the count on the pendulum reaches the neighborhood of fifty, the first listens for a coincidence between the pendulum and the clock, and as soon as one is observed he announces the fact by giving his count aloud. Each records the number of vibrations and the number of seconds counted. They should now change places and repeat the work. Great care must be taken in counting the vibrations, the tendency being to "lose your count" by not keeping the mind fixed on the work. If the computed times for the different trials differ in the third decimal place, the coincidences have not been carefully marked. The error can be reduced by taking a greater number of counts per trial, passing over several coincidences as if they had not occurred. Measure the length of the pendulum from the point of suspension to the top of the ball by setting a carpenter's try-square against a vertical meter rod attached to the support, the blade reaching out to the pendulum. To this length add half of the vertical diameter of the ball obtained by a pair of calipers.

Now, set the same pendulum swinging through an arc several times larger than before and determine the time. Compare the time with that when the arc was small. Why should there be a difference?

Shorten the pendulum and again determine the time, using a small arc. Measure the length of this pendulum. In like manner try a still shorter pendulum, finding both time and length.

Divide the time of each pendulum when the arc was small by the square root of the corresponding length, expressing the result to four decimal places. These quotients should be constant. If so, what law is proved?

Using the data for the longest pendulum and small arc, compute g by the formula $g = \frac{\pi^2 l}{t^2}$. t should be calculated to the fourth decimal place.

Remarks. — One of the principal sources of error in this problem is in judging of coincidences. This error can be obviated by using a pendulum support, as shown in Fig. 34. The frame has a clamp, *A*, at the top, and a second clamp, *C*, on a slider for varying the length of the pendulum. Two meter rods screwed to the support furnish a scale for measuring the length of the pendulum. A carpenter's try-square is used as an index. The clock has an electrical connection, and a telegraph sounder is in the circuit. The pendulum to be tested is also electrically connected through a second sounder, but is in series with the clock, the clock sounder being on a shunt circuit. Hence, the clock sounder speaks at every beat of the clock, but the pendulum sounder speaks only at coincidences of the two pendulums. But one battery of three or four cells in series is needed for both clock and pendulum, and, in fact, half a dozen pendulums can be connected in parallel and all operated at the same time. Since the mercury contact has width, the intervals between coincidences may vary. A certain law of succession will be found to govern them, and the observations should be continued till the cycle of coincidences is obtained, that

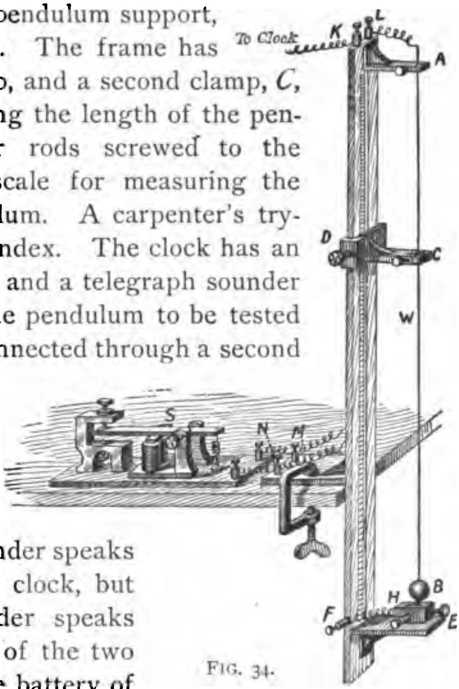

FIG. 34.

is, till the succession of intervals between coincidences repeats itself. Then the time will be the total number of seconds divided by the total number of vibrations comprised in the cycle of coincidences.

Form of Record.—The following form may be used in tabulating the data:—

ARC	NO. OF VIBRATIONS	NO. OF SECONDS	TIME t	DATA FOR l			l	$t \div \sqrt{l}$	g
				Top	Bottom	Diam. of Ball			
Small	—	—	—	—	—	—	—		
	—	—	—	—	—	—	—		
	—	—	—	—	—	—	—		
	—	—	—	—	—	—	—		
	—	—	—	—	—	—	—		
Mean			—				—	—	—
Large	—	—	—						
	—	—	—						
	—	—	—						
	—	—	—						
	—	—	—						
Mean			—						
Small	—	—	—	—	—	—	—		
	—	—	—	—	—	—	—		
	—	—	—	—	—	—	—		
	—	—	—	—	—	—	—		
	—	—	—	—	—	—	—		
Mean			—				—	—	
Small	—	—	—	—	—	—	—		
	—	—	—	—	—	—	—		
	—	—	—	—	—	—	—		
	—	—	—	—	—	—	—		
	—	—	—	—	—	—	—		
Mean			—				—	—	

Note. — If the electrical pendulum is used, then the above form of tabulation should be slightly changed, since there will then be but one calculation for *t* under each case.

V. The Simple Machines.

44. Problem. — *To verify the laws of equilibrium of the lever.*

Apparatus. — A **meter rod**, two **scale pans** of the same weight, a **box of weights**, a **pulley**, and a heavy block of wood. Through the center of the meter rod is a piece of stout knitting needle. Two brass ears with suitable holes through them are fastened to the block of wood. The knitting needle, when inserted in these ears, forms a fulcrum for the lever (Figs. 35, 36). A "slider" of sheet lead on the meter rod is to balance the bar. The scale pans can be attached to the lever by wire loops that slide along on the rod so that the lengths of the arms can be varied.

Directions. — For a lever of the first kind, attach the scale pans at unequal distances from the fulcrum, move the slider till the bar assumes a horizontal position, place weights in one of the pans and then add to the other enough weights to make the bar maintain its horizontal position (Fig. 35). Now find the distance that each pan is from the fulcrum. Compare the ratio of the loads with the inverse ratio of the distances

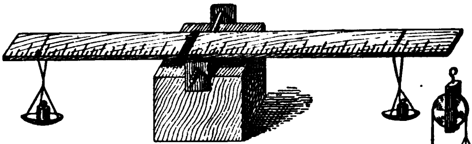

FIG. 35.

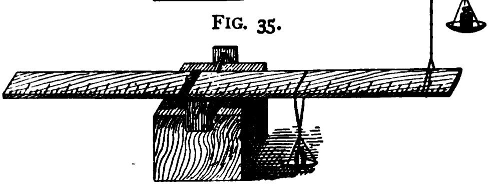

FIG. 36.

or lever arms. Make several changes in the loads and arms and compare as before.

For a lever of the second kind, suspend the pulley from the arm of the iron stand, attach one end of the pulley cord to the end of the meter rod and a scale pan to the other end of the cord (Fig. 36). Adjust the pulley cords to a vertical position. Attach the second scale pan at some point between the fulcrum and the point of attachment of the cord, and move the slider till the bar is horizontal. Now place weights in the pans, and when equilibrium is secured, compare the loads and arms as before. Repeat the experiment several times, varying both the loads and the arms.

For a lever of the third class, exchange the points of attachment of the scale pan and pulley cord in the second case and proceed as before.

Remarks. — On account of friction it will require close observation to determine when equilibrium is secured. A fair degree of accuracy, however, can be obtained by tapping the lever and noticing whether blows of equal intensity applied, first to the top side and then to the under side of the lever, move it through equal arcs.

Form of Record. — The work may be tabulated as follows: —

KIND OF LEVER	WTS. IN LOAD PAN	WTS. IN EFFORT PAN	RATIO	LOAD ARM	EFFORT ARM	INVERSE RATIO	ERROR
——	——	——	——	——	——	——	——
——	——	——	——	——	——	——	——
——	——	——	——	——	——	——	——
etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.

45. Problem. — *To verify the law of equilibrium for any given combination of pulleys.*

Apparatus. — Three brass **pulleys,** two small **scale pans,** cord, fine shot, and a set of **weights.**

Directions. — Set up in succession the combinations shown in Fig. 37. Place shot in one of the pans to counterbalance the weights of the pulleys. Then place known weights in the pans, and ascertain their ratio when equilibrium is secured. Compare this ratio with the number of movable pulleys. Repeat the experiment several times, using different loads each time.

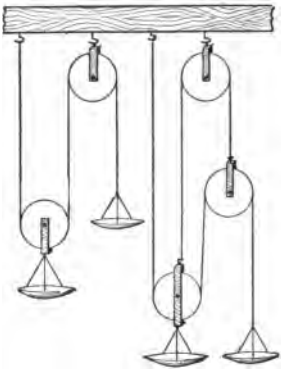

FIG. 37.

Form of Record. — The results may be tabulated as follows: —

	W	W'	$W \div W'$	THEORETICAL RATIO	ERROR
First trial . .	——	——	——	——	——
Second " . .	——	——	——	——	——
etc.	etc.	etc.	etc.	etc.	etc.

46. Problem. — *To verify the laws of equilibrium of the wheel and axle.*

Apparatus. — A **special device,** consisting of a number of cylinders differing in diameter and all turning together on a common axis (Fig. 38), fine shot, a set of **weights,** and a pair of **calipers.** A cord is attached to each cylinder to support a scale pan.

Directions. — Attach scale pans to two of the cords, and place fine shot in one till equilibrium is secured. This will neutralize the effect of the pans. Measure the diameter of each cylinder. Now place known weights in the pans, and ascertain their ratio whenever equilibrium is secured. Compare this ratio with the inverse ratio of the diameters of the cylinders used. Make several trials, using the different cylinders.

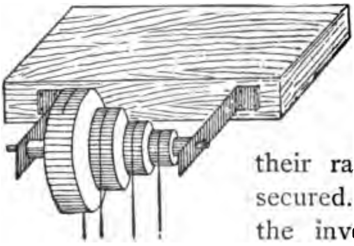

FIG. 38.

Remarks. — Equilibrium may be considered as secured when the scale pans move alike, up or down, on giving them a thrust with the hand.

Form of Record. — The results may be tabulated as follows: —

	D_1	D_2	W_1	W_2	$D_2 \div D_1$	$W_1 \div W_2$	ERROR
First trial .	——	——	——	——	——	——	——
Second " .	——	——	——	——	——	——	——
etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.

47. Problem. — *To verify the law of equilibrium of the inclined plane when the force is applied parallel to the plane.*

Apparatus. — A **narrow board**, about 1.5 m. long, a heavy plate of glass, of the same length and width, a short **iron cylinder**, weighing about 2 kgm., and provided with a bail. A small pulley should be attached either to one end of the board, or to an adjustable support. A stout, flexible **cord**, a **scale pan**, a set of **weights**, and a **block** or **brace** to support one end of the board, will be necessary.

Directions. — Support one end of the board so that the surface makes an angle of about 30° with the table, and place the plate of glass upon it. Weigh the iron cylinder. Fasten a cord to the bail of the iron cylinder, and bring it over the pulley attached to the upper end of the board, the cylinder resting on the inclined surface (Fig. 39). The pulley should be so adjusted that the cord, when drawn taut, is parallel to the surface of the board. Attach the scale pan, having determined its weight, to the end of the cord as it hangs over the pulley, and then place in it weights till the number is sufficient to cause the cylinder or roller to move steadily and slowly up the incline. Record the total load moved and the total force that is moving it. Now remove from the pan enough weights so that the cylinder descends the plane at a speed, as nearly as can be estimated, equal to that at which it previously ascended. The weight of the pan plus the weights in it is the value of the force that is pulling on the cord. Repeat this operation at least five times, recording the results each time. The means of these results will be the force applied when the cylinder or load is ascending the plane, and that when the cylinder is descending the plane, re-

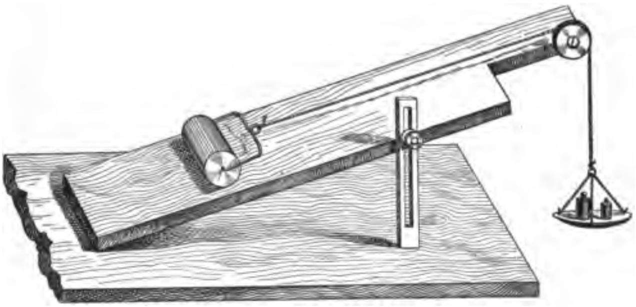

FIG. 39.

spectively, and from these the true force can be computed. If F be the true force required to move the load up the plane, provided there were no friction, and if f be the friction, then $F+f$ will be the total force applied when the load ascended the plane, and $F-f$ will be the force actually applied when the load descended the plane. The mean of these two values will be the force required to move the load on the plane if there were no friction. Now find the length and the height of the plane. To find the latter, adjust, by the aid of a spirit level, a straightedge on the table by the side of the inclined plane so that it is horizontal; then measure the height that the inclined surface at each end is above this straightedge. The difference between these heights will be the height of the plane. Compute the ratio of the total load to the mean force, and also the ratio of the length of the plane to the height. Make several trials.

Remarks. — The length and height of the plane must be measured with the greatest care. If the surface of the board is hard and highly polished, the glass may be omitted.

Form of Record. — The results may be tabulated as follows: —

Length l	Height h	Wt. of Pan	Wt. in the Pan	Load L	Force up or $F+f$	Force Down or $F-f$	True Force F	$\frac{l}{h}$	$\frac{L}{F}$	Error
—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—
etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.

48. Problem. — *To measure the coefficient of friction between two sliding surfaces.*

Apparatus. — A **smooth pine board** about 1 m. long and 20 cm. wide, a rectangular **pine block** 10 cm. by 5 cm. by 2 cm., a **pulley, cord, scale pan,** and **weights.**

Directions. — Weigh the rectangular block, and then fasten the cord to the center of the end, pass it over the pulley fixed to the end of the pine board, resting horizontally on the table, and attach the scale pan to the free end. The pulley must be adjusted so that the cord is horizontal. Place weights on the top of the block till the total mass is 100 gm. Then find by trial the least weight that must be placed in the pan to start the block. Repeat at least three times and find the mean. Increase the load by 50 gm. and repeat as before. Continue in this manner till the load reaches 1 kgm. The moving force will be the sum of the weights in the pan and the weight of the pan. The coefficient of friction is the ratio of the moving force to the load moved. Compute this coefficient for each load, and find the mean.

Plot a curve in which the forces and loads are the ordinates and abscissæ respectively.

Repeat the work, placing the block on its smallest face, thus making the surface in contact a fifth of what it was in the first case. What effect does the area of the surface have?

Rub the surfaces in contact with chalk and repeat the trials. Plot the results as before.

Clean off the chalk and rub the surfaces with paraffin. Repeat the trials and plot the results.

Remarks. — In discussing this problem, the effects of load, area of surfaces in contact, and the character of these surfaces should be considered. The problem can be varied indefinitely by trying different kinds of wood, by moving the slider crosswise of the grain, or by coating the surfaces with thin sheets of different substances. The friction of the pulley introduces a small error.

Form of Record. — The results may be tabulated as follows: —

WT. OF PAN	WTS. IN PAN	MOVING FORCE f	LOAD W	COEF. OF FRICTION μ
——	——	——	——	——
——	——	——	——	——
etc.	etc.	etc.	etc.	etc.
			Mean . .	——

49. Problem. — *To measure the angle of repose.*

Apparatus. — Same as in Art. 48.

Directions. — Find the weight of the block, place it on the board, and add weights till the pressure is 100 gm. Then gently raise one end of the board until the block begins to slide. Measure carefully the length of the board and the height of the end above the supporting table. The quotient of the latter by the former will be the sine of the angle of repose. Repeat the experiment with the load increased successively by 50 gm., and obtain the mean of these sines. The value of the angle can be found by means of Table XIX of the Appendix. What effect does the load have on the angle? Compare the angle with the angle that the curve of Art. 48 forms with the axis of X. Compare the tangent of the angle of repose with the coefficient of friction given by Art. 48.

Remarks. — This problem can be varied in the same manner as in Art. 48. The weights can be kept on the sliding block by cementing a paper box to the top of the block.

Form of Record. — The results may be tabulated as follows: —

LOAD MOVED W	HT. OF PLANE h	LENGTH OF PLANE l	$\frac{h}{l}$	ANGLE
——	——	——	——	
——	——	——	——	
etc.	etc.	etc.	etc.	
		Mean . .	——	——

CHAPTER IV.

MECHANICS OF FLUIDS.

I. Surface Tension and Capillary Action.

50. Problem. — *To determine the conditions that govern drop size.*

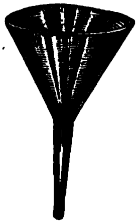

FIG. 40.

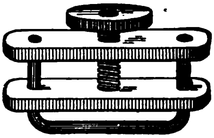

FIG. 41.

Apparatus. — A **balance** weighing to centigrams, two **air thermometer bulbs** of different diameters, a **funnel** (Fig. 40) with a short piece of rubber tubing attached to its stem, an adjustable **pinchcock** (Fig. 41), an **iron ring stand** (Fig. 70), and a **universal wooden stand** (Fig. 42).

Directions. — Set up the apparatus as shown in Fig. 43. Close the rubber tube on the stem of the funnel by means of the pinchcock. Fill the funnel two thirds full of the liquid to be tested. Adjust the pinchcock so that the liquid flows from the funnel upon the glass bulb below it, spreads evenly over the bulb, and falls in drops from its under surface. To secure this result the glass globe must be clean. To clean the globe wash it in a solution of caustic potash and then rinse it in distilled water.

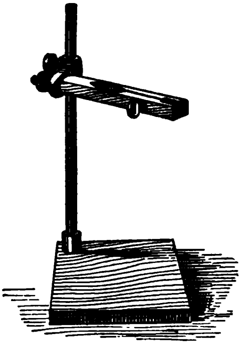

FIG. 42.

First. By trial adjust the pinchcock so that the drops fall regularly at the rate of one per second. As soon as this rate is secured catch fifty drops in a small glass beaker of known weight and weigh them. Divide the weight of the fifty drops expressed in grams by the density of the liquid (Table II of the Appendix) and the quotient will be their volume in cubic centimeters. This volume divided by fifty will be the volume of a single drop. Make at least three trials and find the mean. Now adjust the pinchcock so that two drops fall per second. Then catch fifty of these drops and find the volume of a drop as in the first case. A comparison of these two sets of results will show the effect of drop rate on volume.

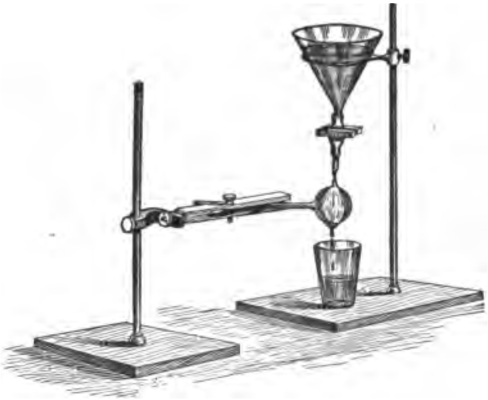

FIG. 43.

Second. Substitute the other glass globe and find the volume of a drop as before, using the same liquid, the drop rate being set at one drop per second. Compare this result with that of the first case and the effect of shape of surface will be shown.

Third. Keep the conditions the same as in the second case, except that a different liquid is used, and determine the drop size. A comparison of this result with that of the second case will show the effect of the cohesion of the liquid on drop size.

Remarks. — Volatile liquids, as alcohol, ether, etc., are seriously affected by evaporation. Suitable liquids for use are water, glycerine, sweet oil, and acetic acid. What effect should a high density have on the drop size? A more uniform flow of the liquid from the funnel can be secured if a short glass tube is inserted in the rubber tube and the end is ground off obliquely. It is also necessary to keep the liquid at nearly a constant level, since a change of pressure will produce a change of flow and destroy the uniformity of the drop rate.

Form of Record. — The results may be tabulated as follows: —

LIQUID USED	SIZE OF SPHERE	DROP RATE	WEIGHT OF VESSEL	WEIGHT OF VESSEL AND FIFTY DROPS	WEIGHT OF FIFTY DROPS	VOLUME OF FIFTY DROPS	DROP SIZE
				—			
				—			
				—			
—	—	—	—	—	—	—	—

51. Problem. — *To verify the laws of capillary action.*

Apparatus. — Several **capillary tubes** whose diameters are known (24, 33), a glass **beaker**, linear **steel scale** graduated to millimeters and one end ground to a V-shape, the zero of the scale being the point of the V (Fig. 44).

Directions. — Clean the tubes by washing them, *first*, with hydrochloric acid, and, *secondly*, with distilled water, as directed in Art. 33. It may be necessary with some tubes to place them in a test tube containing nitro-muriatic acid and heat them till the liquid boils up through the tube in order to get them clean. Attach one of them to the scale with rubber bands so that about one centimeter of the tube projects beyond the point of the V. Support the scale in a vertical position with the point of the V just touching the surface of the water in the beaker. Now lift the beaker upwards so that the capillary tube may be wet several centimeters up the tube, then lower it to its first position, see that the zero of the scale is in exact contact with the surface of the water, and after waiting a minute to make sure that the water has fallen in the tube to its proper level, read the height of the column on the scale, using a pocket microscope for the purpose. The reading should be taken to the bottom of the meniscus and in a line normal to the scale. Repeat the reading several times, always raising the beaker to wet the tube and carefully adjusting the zero of the scale before each reading. Correct the mean of these heights for the meniscus, by adding to it one sixth of the diameter of the tube. Multiply each corrected height by the diameter of the corresponding tube. These products will be nearly equal, showing that the height varies inversely as the diameter of the tube.

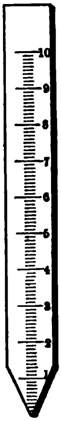

FIG. 44.

Find the height to which water will rise in any one of the tubes. Compare this height, when corrected for the meniscus, with-that obtained with this tube for cold water. What effect does temperature have?

Find the height to which such liquids as grain alcohol, wood alcohol, ether, turpentine, etc., rise in any one of the tubes. Compare these results with that obtained for water with the same tube.

Remarks. — The tubes must be thoroughly cleaned after each liquid. Air bubbles in the tube will prevent the liquid's rising to its proper height, but *do not blow through the tube to remove them.* Either jar them out or insert a very fine wire. The data obtained for cold water should be represented by a curve (15), the diameters of the tubes and the corrected heights being the coördinates. The curve is known as the equilateral hyperbola.

Form of Record. — Tabulate the heights to which the liquid rises in each tube in several parallel columns. Summarize the mean results into a table of the following form:—

DIAM. OF TUBE	HEIGHT WATER RISES	HEIGHT CORRECTED FOR DIAM.	$H \times D$	HEIGHT FOR HOT WATER	HEIGHT FOR ALCOHOL	HEIGHT FOR ———	HEIGHT FOR ———
——	——	——	——	——	——	——	——
——	——	——	——				
——	——	——	——				
——	——	——	——				
——	——	——	——				

II. Liquid Pressure.

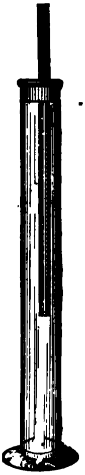
FIG. 45.

52. Problem. — *To verify the laws of liquid pressure.*

Apparatus. — Two **metal tubes**, preferably aluminium, one about 1.5 cm. in diameter, and the other 2 cm., each tube 50 cm. long, and closed watertight at one end; a number of 5-gm. **weights** or **lead balls** of equal and known mass; a quantity of shot; a deep **cylindrical glass vessel** filled with water (Fig. 45). Each tube should have a linear scale extending its entire length, the zero of the scale being at the closed end. The tubes should have thin walls to make them as light as possible. Paraffined paper tubes can be used instead of metal

Directions. — *First.* Load the smaller tube with shot till it floats vertically in the jar of water. Record the depth to which the tube sinks in the water. Weigh the tube and shot. Now drop 5-gm. weights in the tube, recording the level of the water on the tube and the total load as each weight is added. Continue adding weights till the tube is nearly submerged. Divide the total load for each depth by the depth. What law do the results suggest? Represent the data by a curve (15). What does the curve show?

Second. Load both tubes with shot till they sink to exactly the same depth in water. Weigh each tube and its load of shot. Measure the outside diameter of each tube and compute the area of the base of each. Compare the ratio of these areas with that of the weights.

Third. Fill the cylindrical vessel with a saturated solution of salt. Load one of the tubes with shot till it floats at the same depth in the salt solution as it does in water. Find the density of the salt solution, if it is not already known (58). Compare the ratio of the loads that correspond to equal submergence in water and salt solution, respectively, with the ratio of the densities of these liquids.

Remarks. — Such liquids as alcohol and kerosene can be used in place of the salt solution. If a bottle of known weight be filled with water to a mark cut on the neck with a file and weighed, and afterwards with the liquid to the same point and weighed, the ratio of these weights after deducting that of the bottle will be the same as the ratio of the densities. By *load* is to be understood the combined weight of the tube, the contained shot, and the weights.

Form of Record. — The results may be tabulated as follows: —

Load for Small Tube L	Depth submerged H	$\frac{L}{H}$
——	——	——
——	——	——
——	——	——
——	——	——
——	——	——
——	——	——
——	——	——
etc.	etc.	etc.

Diameter of Small Tube	Diameter of Large Tube	Area of Base of Small Tube, A	Area of Base of Large Tube, B
——	——		
——	——		
——	——		
——	——	——	——

Loads when Tubes are equally submerged in Water

Small Tube L	Large Tube L'	$\frac{L}{L'}$	$\frac{A}{B}$
——	——	——	——

Loads when tube is equally submerged in Water and in Salt Solution

L for Water	L' for Salt Solution	Density of Salt Solution, or D	$\frac{L'}{L}$	$\frac{D}{d}$
——	——	——	——	——

III. Boyle's Law.

53. Problem. — *To show that the volume of a given quantity of a dry gas varies inversely as the pressure, the temperature being constant.*

Apparatus. — A **device** made as follows: Two glass tubes, A and B, are connected by a stout rubber tube and attached to a wooden support, which can be clamped to a table (Fig. 46). The tube, *A*, should have a very uniform bore of about 5 mm. Its free end should be plugged with a heavy walled capillary tube of not over 1 mm. bore, the length of the plug being 2 cm. The plugged end should be ground to a plane. An iron ferule on the end of the tube, and fitted with a cap and leather washer, will make it possible to close the tube air-tight. The ferule should be a trifle shorter than the glass plug, so that the lower end of the plug can be seen without removing the ferule. The lower end of the glass plug will be practically the top of the air column inclosed, since the quantity in the capillary part may be neglected without sensible error. The tube, *B*, is left open, and may be of larger diameter than *A*. Each tube is fastened to a wooden strip that slides freely in a groove by the side of a linear scale. An arm attached to each slider makes it possible to fasten it at any point on the scale by means of a stout pin inserted in one of the series of holes in the frame. The rubber tube must be sufficiently heavy so that it will not stretch when filled with mercury. It should be lined to prevent the action of the sulphur of the tube on the mercury. To do this, fill it with a thick solution of gutta-percha in chloroform. Let it stand a few minutes, then empty, and suspend in a vertical position. It is advisable to blow through the tube after suspending it to prevent the solution from clogging it as it drains out. A thermometer should be attached to

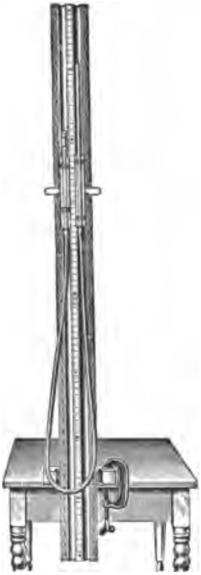

FIG. 46.

the frame, and a barometer should be near at hand on the wall.

Directions. — Fasten *A* and *B* at about the middle of the linear scale. Remove the cap from *A*, and by means of a small funnel pour mercury in *B* till *A* is filled about half way up, repeatedly jarring the tube to drive out all air bubbles. Raise *B* till *A* is full, the air in *A* being all driven out. Connect *A* to a gas-drying bottle, using a rubber tube for the purpose; then lower *B* to its original position, thereby dragging dry air into *A*. Before attaching the rubber tube to *A*, however, it will be necessary to force dry air through the tube to remove the moist air it already contains. Now close *A* with the cap and lower *B* as far as the scale permits. This expands the air in *A*, and in so doing it is cooled. Hence, one must wait several minutes for it to return to the temperature of the room. A T-square shaped slider placed against the glass tube is used in reading the position of the mercury. Record the reading of the top of the air column in *A*; also the top of the convex surface of the mercury in *A*, the top of the convex surface of the mercury in *B*, the reading of the barometer, and that of the thermometer. The air in *A* will be under a pressure obtained by adding the barometric pressure to the reading of the mercury in *B*, and subtracting from the sum the height of the mercury in *A*. The length of the air column in *A* is the difference between the reading of the top of *A* on the scale and the top of the mercury in *A*. Since the tube *A* is of uniform bore, the volume of the inclosed air will vary as the length, and hence length may be substituted for volume in testing the law.

Now move *B* upward one point on the support, say 10 cm., and repeat the readings and computations. Proceed in this way till *B* reaches the top of the support. After each movement of *B*, several minutes must be allowed for the compressed air to cool, before the position of the mercury is read.

Find the product of the length of the air column by the corresponding pressure. If these products are nearly constant, what law is suggested? Plot a curve, using as coördinates the lengths of air column and the corresponding total pressures (15). To what class of figures does the curve belong? Using the first air column and the corresponding total pressure as a basis, compute what the air column should be for each pressure. Find the error of each observation of the air column.

Remarks. — It is very important that the temperature of the room does not change while performing the experiment. An increase in temperature would cause an increase in the product quantity, $V \times P$, whereas a decrease in temperature would cause a decrease in these products. The mercury used should be dry and clean. Avoid contact in any way with the rubber tube when taking readings, since any pressure upon it would change the readings.

Form of Record. — The results may be tabulated like the following: —

READING OF *A*	READING OF *B*	READING OF THER.	READING OF BAR.	LENGTH OF AIR COL., *V*	TOTAL PRESSURE, *P*	$V \times P$	COMPUTED AIR COL.	ERROR
cm.	cm.		cm.	cm.	cm.		cm.	cm.
119.45								
90.50	67.20	20°.6 C.	74.18	28.95	50.88	1473	28.95	0.00
92.64	73.60	20°.6 C.		26.81	55.14	1478	26.71	0.10
94.60	79.90	20°.6 C.		24.85	59.48	1478	24.76	0.09
96.40	86 40	20°.6 C.		23 05	64.18	1479	22.95	0.10
etc.	etc.			etc.	etc.	etc.	etc.	etc.

IV. Siphon.

54. Problem. — *To prove that the rate of flow of a liquid through a siphon is proportional to the square root of the difference of lengths of the arms.*

Apparatus. — A **siphon**, a large vessel of water, a **graduate**, and a **meter rod**. The siphon may be made by bending a glass tube of at least 6 mm. bore, twice at right angles, making the branches equal and parallel. The branches should be about 50 cm. long and the vessel of water at least 40 cm. deep. Arrange the vessel, or tank, containing the water, so that there is a steady flow of water into it; then by means of an overflow pipe, the water can be maintained at a constant level.

Directions. — Support the siphon with one branch in the vessel of water, with the part connecting the arms horizontal, and about 10 cm. above the water. Measure the length of the arms, the short one being the vertical distance of the highest point of the siphon above the water, and the long one being the vertical distance of the highest point above the outlet. Start the siphon by suction, and measure the amount of flow in one minute or any other convenient unit of time. Raise the siphon a few centimeters, thereby increasing the length of the short arm, and repeat the measurements. Secure at least five changes in the length of the short arm. Divide the amount of flow per minute by the square root of the corresponding difference of the arms. These quotients will be nearly constant, if the experiment is successful. Taking the first observation as a basis, compute by the law what the flow should be for the successive arms used. Why do the observed and theoretical flow differ? Plot a curve using the observed amount of flow, and excess of long arm as coördinates (15). What should the curve be if the square root of the difference of the arms be used?

Remarks. — In discussing this problem, show, *first*, that by the principle of fluid pressure the flow should be greater, the greater the difference between the arms; *secondly*, that by the laws of falling bodies the flow should vary as the square root of this difference and not as the difference; *thirdly*, that the actual flow should be less than the theoretical flow.

Form of Record. — The results may be tabulated as follows: —

LONG ARM, L	SHORT ARM, l	$L-l$ OR D	FLOW IN 1 MIN., V	$V \div \sqrt{D}$	COMPUTED V	DIFFERENCE
——	——	——	——	——	——	——
——	——	——	——	——	——	——
——	——	——	——	——	——	——
etc.	etc.	etc.	etc.	etc.	etc.	etc.

V. The Principle of Archimedes.

55. Problem. — *To show that the buoyant force of a liquid on a submerged body is equal to the weight of the liquid that the body displaces.*

Apparatus. — **Balance, weights, vernier caliper, thermometer,** and a solid **metal cylinder** with its ends perpendicular to the axis of the cylinder are needed for this problem. The cylinder may be cut from a metallic rod; it should be smooth, a true cylinder, and need not exceed 1.5 cm. in diameter and 2 or 3 cm. long.

Directions. — Measure the length and diameter of the cylinder several times with a vernier caliper (23). From the mean of these measurements compute the volume of the cylinder in cubic centimeters (see Table XV of the Appendix). Find the weight of the cylinder as directed in Art. 31, expressing the result in grams. Now tie a fine thread to the cylinder, and suspend it from the small hook usually found at the top of the bail of the balance pan (Fig. 47). Place a small wooden bench astride of the balance pan, so as not to interfere with the action of the balance. On this bench place a beaker of water whose temperature is that of the room. The thread supporting the cylinder should be very carefully adjusted as to length, so that the cylinder will be always entirely submerged during the process of weighing it. Now find the weight of the cylinder while submerged in water, and compute the loss in weight. Ascertain exactly the temperature of the water, and compute by means of Table III of the Appendix, from the volume displaced, the weight of the water displaced. Find the difference between the loss in weight as given by the balance and the computed weight of the water displaced. Compute the per cent. of error, using the computed weight as the base. Tabulate the data given by the balance as directed in Art. 31. At the end collect into a table all the results as follows:—

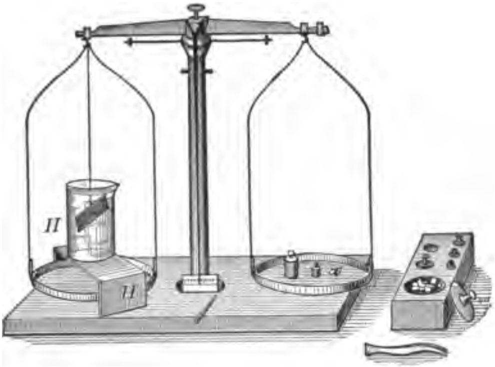

FIG. 47.

Vol. of Cylinder Measured	Weight of Cylinder in Air	Weight of Cylinder in Water	Buoyancy of Water	Computed Weight of Water Displaced	Error	Per Cent. of Error
——	——	——	——	——	——	——

Remarks. — Wrap the thread but once around the cylinder and cut off all long ends. Remove with a wire all air bubbles adhering to the cylinder when in water. The surface tension and viscosity of the water will interfere with the balance's freedom of vibration. If so, it may be necessary to find by trial what weights it would be necessary to place in the pan to bring the pointer to the true resting point instead of computing the correction. In discussing this problem, show from theoretical considerations what should be the magnitude of the buoyant force. What are the chief sources of error?

VI. Determination of Density or Specific Mass.

56. Problem. — *To find the density of a body heavier than water.*

Apparatus. — **Balance,** set of **weights, thread, beaker,** and **thermometer.**

Directions. — Suspend the body by a fine thread from the hook on the bail of the scale pan, and accurately weigh it. Then weigh it submerged in distilled water as described in Art. 55. The difference of these two weights is the mass of the water displaced. (Why?) Take the temperature of the water and find its density from Table III of the Appendix. Divide the mass of the water displaced by the density of the water; the quotient will be the volume of the water displaced, and hence the volume of the body. Now divide the mass of the body by its volume, and the quotient will be the density.

Remarks. — All adhering air bubbles must be brushed from the submerged body. If the solid is soluble in water, then it must be weighed submerged in some liquid in which it is not soluble, and the density of that liquid takes the place of the density of water in the computation.

Form of Record. — Tabulate the data after the plan pointed out in Art. 55.

57. Problem. — *To find the density of a body lighter than water.*

Apparatus. — Same as in Art. 56.

Directions. — Weigh the body in air as in Art. 56. Then suspend from the hook or scale pan, so as to hang in water, a body of sufficient mass to make the given body sink in water, if attached to it. Find the combined weight of the two bodies, the given body on the scale pan, and the sinker suspended in water. Attach the body to the sinker and find the weight of both submerged in water. The difference between the last two results will be the weight of the water displaced by the body. (Why?) Now complete the problem as directed in Art. 56.

Remarks. — A sinker provided with a sharp point is a convenient form, for it can be attached to the body by thrusting the point into it.

Form of Record. — Tabulate the data after the plan pointed out in Art. 55.

58. Problem. — *To find the density of a liquid.*

liquids

hydrom

Apparatus. — **Balance, weights, density bottle, Y-tube, glass tubing, beakers, pinchcock** (Fig. 41), etc. The density bottle (Fig. 48) is a small flask, holding 10, 25, 50, or 100 cm.^{3}, closed with an accurately fitting glass stopper with a capillary opening.

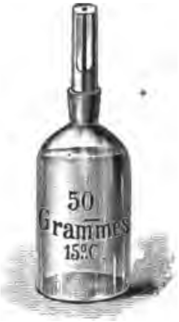

FIG. 48.

Directions. — *Density Bottle Method.* — Thoroughly clean the density bottle by rinsing it out successively with hydrochloric acid, water, and alcohol. Dry it in a current of warm air, or by inserting a glass tube draw air through the bottle by suction. Find the weight of the empty bottle (31). If the volume of the bottle is not known, fill it with distilled water, introduce the stopper carefully, without pressure, giving the surplus water time to escape through the perforation in the stopper. Dry the outside of the bottle with a soft cloth. Avoid heating the bottle in handling. Find the weight of the bottle and contents; deduct from this weight that of the bottle; the remainder will be the weight of the water, from which the volume of the bottle is easily obtained. (How?) Dry the bottle and then fill with the liquid whose density is sought, and find the weight. This weight, less that of the bottle, divided by the volume of the bottle, will be the density.

Weighing Body in Liquid. — Procure a glass stopper or some heavy solid on which the liquid has no action. Weigh the stopper in the air and then in water. From these data calculate its volume (55). Weigh the stopper in the liquid and calculate the mass of the liquid displaced by it. Divide this mass by the volume of the stopper and the quotient will be the density of the liquid.

Hare's Method. — Connect two glass tubes, each 30 cm. long, to the branches of a Y-tube, also connect a rubber tube to the stem. Support the apparatus vertically with the open ends of the glass tubes dipping respectively into two beakers or tumblers, one containing water and the other the liquid whose density is sought (Fig. 49). Measure with a linear scale the capillary effect and record. Remove, by suction, part of the air, closing the rubber tube by means of a pinchcock. Atmospheric pressure will force the liquids up their respective tubes. Measure the vertical height of the liquids above the level in their respective vessels, and subtract that due to capillarity. Divide the height of the water by that of the liquid, and the quotient will be their relative densities. From the temperature of the water find by Table III of the Appendix its absolute density. This multiplied by the relative density of the liquid will be the density at the temperature observed.

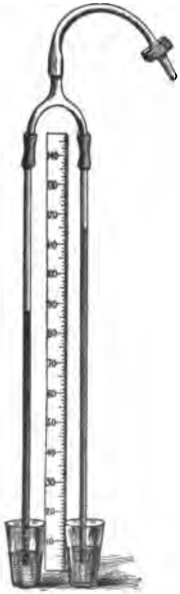

FIG. 49.

Form of Record. — Previous problems in weighing and measuring will suggest forms of tabulation.

CHAPTER V.

SOUND.

I. Reënforcement of Sound.

59. Problem. — *To find what correction must be made for diameter in the length of a cylindrical resonating air column.*

Apparatus. — A **tuning fork** whose vibration rate is accurately known, C = 256 is preferable; a set of three **cylindrical tubes**, brass or zinc, of diameters about 2 cm., 3.5 cm., and 5 cm., respectively, and 40 cm. long; a pair of **inside calipers**, a **thermometer**, a **linear scale**, and a vessel of water about 20 cm. deep, the mouth of which is large enough to admit any one of the tubes. Figure 50 shows the apparatus mounted for use.

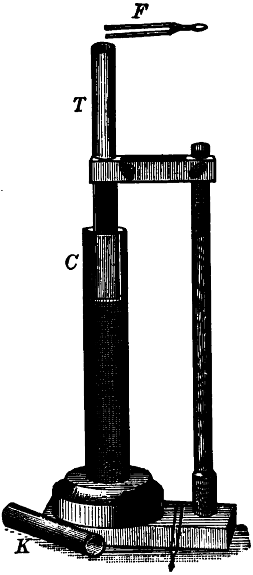

FIG. 50.

Directions. — It is shown in theoretical physics that in a cylindrical resonating air column the actual length is always less than the true length, and a correction to the observed length, depending on the diameter, should be made for the open end. To find this correction, measure the inside diameter of each tube with the inside calipers (25).

Hold one of the tubes in a vertical position with the lower end dipping into the water. Set the fork in vibration by striking one corner of one of its prongs sharply against a piece of wood. Hold the ends of the prongs over the open end of the tube, and at the same time alternately raise and lower the tube, noting the place of the tube in which the reënforcement is loudest. Make certain that this place gives the loudest resonance by testing points on each side, and comparing them with this one. Measure the length of this air column by holding the linear scale by the side of the tube, the end just touching the water. Ascertain the temperature of the air by suspending the thermometer within the tube for a few minutes. Repeat the work several times with each tube, and find the mean length of the air column for each tube. Calculate the velocity of sound in the air, using the relation $v = 332 + 0.6\,t$, where t is the temperature on the centigrade scale, and 332 is the velocity in meters at 0° C.

If n is the vibration rate of the fork, and l the wave length, then $l = v \div n$; $\frac{1}{4}\,l =$ the length of the shortest resonating column. The observed length, when carefully obtained, will be found invariably shorter than the computed length. The difference divided by the diameter of the corresponding tube will be the correction ratio. When the work is carefully done, these ratios for the three tubes will be found practically the same, and the mean of these may be taken as the correction ratio for tubes whose diameters are within the limits used in the experiment.

Remarks. — It will take considerable preliminary practice on the part of the student to be able to make such fine distinctions of loudness as will insure success. Hold the fork firmly by the stem, and be careful not to dampen it by touching the shoulders of the prongs. The line of the ends of the prongs must be parallel to the mouth of the tube, and at a distance not to exceed a centimeter above the plane, to secure the best reënforcement.

By using for this experiment tubes whose lengths are a little over three fourths of a wave length, the correction for diameter can be found as follows: Find by trial the shortest air column that reënforces the fork and also the next shortest column. The first will be a quarter of a wave length and the second will be three quarters of a wave length. The difference will be a true half-wave length, since the correction for diameter belongs to the open end and is eliminated by subtracting. Half of this difference will be a true quarter of a wave length. The difference between the observed quarter and the true quarter divided by the diameter will be the correction ratio.

Form of Record.—Tabulate as in the following example:—

N.	DIAM.	LENGTH OF AIR COL.	TEMP.	VEL. OF SOUND	COMPUTED LENGTH OF AIR COL.	DIFFERENCE	RATIO
256	2.0 cm.	32.54 cm. 32.70 " 32.80 " 32.68 "	19° C.	343.4 m.	33.54 cm.	0.86	0.43
256	4.0 "	32.06 " 31.50 " 32.20 " 31.92 "	19° C.	343.4 "	33.54 "	1.62	0.40
256	4.5 "	31.90 " 31.70 " 31.60 " 31.73 "	19° C.	343.4 "	33.54 "	1.81	0.40
						Mean	0.41

II. Velocity of Sound.

60. Problem. — *To measure the velocity of sound in air.*

Apparatus. — A **tuning fork**, preferably $C = 256$, a **thermometer**, and a **glass tube** at least one meter long, and about 3 cm. diameter, fastened to a frame so as to be supported vertically by clamping to the table (Fig. 51). The lower end of the tube is closed by a rubber stopper, single apertured, through which passes a glass tube bent at right angles and connected by a rubber tube, about 1.5 m. long, to a funnel. A small glass tube hooks into the open end of the large glass tube, and to it is connected a listening tube such as is used on an Edison phonograph. A meter scale fastened by the side of the large tube completes the device.

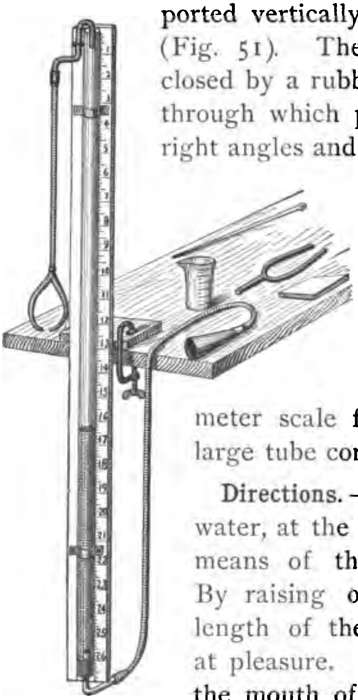

FIG. 51.

Directions. — Fill the tube part full of water, at the temperature of the room, by means of the funnel and rubber tube. By raising or lowering the funnel, the length of the air column can be varied at pleasure. Hold the vibrating fork over the mouth of the tube, then change the length of the air column, and with the listening tube in position, read from the scale the level of the water as the sound swells out to its loudest resonance. Now find a point lower down on the tube where a reënforcement is secured. Since the first reënforcement is produced by a column whose length is $\frac{1}{4}\,l$, and the second by one whose length is $\frac{3}{4} l$, then the difference is $\frac{1}{2} l$, and any correction due to diameter is eliminated (59). Twice this difference will be the wave length, and v can be found from the relation $v = nl$.

Suspend the thermometer in the tube and measure t. Then the velocity in meters derived from the temperature will be $v = 332 + 0.6 t$. This may be regarded as being very near to the true velocity, and may be used as such in computing the per cent. of error.

In case a resonating tube is used that is not long enough to give a second reënforcing column, the shortest resonating column must be increased by two-fifths of the diameter (59) to give $\frac{1}{4} l$. Then as before $v = nl$.

Remarks. — To insure success, the fork should be made to vibrate each time as nearly as possible with the same energy. The reënforcement by the longer column is not as clearly marked as that by the shorter. The listening tube is not absolutely essential. Its advantage is that it eliminates the disturbance due to other pupils working on this problem at adjoining tables.

Form of Record. — The results may be tabulated after the following plan: —

VIBRATION RATE OF FORK = . . . PER SECOND.

FIRST COL.	SECOND COL.	$\frac{1}{2} l$	TEMP.	DIAM. OF TUBE
—— cm.	—— cm.	—— cm.	——	—— cm.
——	——			
——	——	$v = nl =$ . . . . m.		
etc.	etc.	$v = 332 + 0.6 t =$ . . . . m.		
		Error = ... m.		
——	——	Per cent. of error = ...		

61. Problem. — *To find the velocity of sound in a metal rod.*

Apparatus. — A **glass tube** about 1 m. long and 4 cm. in diameter, closed at one end by a membrane of thin sheet rubber tied firmly over it (Fig. 52). To the other end is fitted a cork piston that slides freely in the tube, steadiness being secured by letting the stem slide through a cork inserted in the end. Within the tube is scattered some very fine cork dust made by filing a piece of cork and screening the filings through a fine sieve. A metal rod about 1.5 m. long and 1.25 cm. diameter has a paper disk of 3 cm. diameter cemented across one end. This rod is clamped at its center on the table by means of an iron clasp, *B*, and a cabinetmaker's clamp.

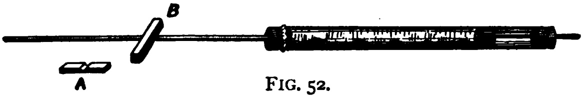

FIG. 52.

Directions. — If a rod of uniform diameter is clamped at the middle, and one half of it is stroked longitudinally with a cloth or soft piece of leather previously dusted with powdered rosin, it emits a sound of high pitch whose wave length is twice the length of the rod, since the clamped point must be a node and each free end a ventrum, and the distance from a node to the adjacent ventrum is one fourth of a wave length. If the vibrations of this rod are communicated to the rubber membrane and by it transmitted to the air within the glass tube, there is produced within this tube a succession of waves which travel through the inclosed air to the piston and are reflected so that each one interferes with the succeeding waves. Now it is evident that, if the air column has the right length, the interference of the advancing and the reflected waves will result in changing the progressive waves into stationary ones, the air vibrating in segments like the air of a cylindrical resonating column, the distance between the nodes depending on the velocity of sound in air and the pitch of the note. The distance, then, between two nodes will be half of a wave length in air for the sound emitted by the rod. If we represent this by s, $2s$ will be the wave length of this sound in air. If d is the length of the rod, $2d$ will be the wave length of the same sound in the rod, and $2d \div 2s$, or $d \div s$, will be the number of times faster that the sound travels in the rod than in the air.

The velocity of sound in air of the temperature, t, is expressed quite accurately by the formula $v = 332 + 0.6t$; and hence the velocity of sound in the rod, or V, will be $= \frac{d}{s}(332 + 0.6t)$.

Hence, to find the velocity of sound in a metal rod, stroke it with the rosined leather, applying gentle pressure, holding the leather between the thumb and first finger of the two hands. The cork dust will be vigorously agitated, rising up in parallel layers, and falling into clearly marked heaps of uniform length, subdivided into parallel ridges, the nodes being indicated by circles of dust. When the piston is properly adjusted there will be a node at each end of the air space in the tube, and the parallel ridges of dust will be sharply defined. The piston must be moved to and fro till such a condition is secured. Measure with the meter rod the length of the air column, count the number of segments; the quotient of the former by the latter will be the value of s. Measure the length of the rod. This will be d. Observe the temperature, t, and compute the velocity of sound in air. This multiplied by $\frac{d}{s}$ will be the velocity of sound in the rod. In repeating the work the piston should be moved several centimeters, and then brought by trial to the position in which the nodes are most clearly defined.

Remarks. — In stroking the rod the pressure should be gentle, the movement quick, and in exact line with the length of the rod. The proper pressure for the rod against the membrane must be found by trial. Mean values of t, d, and s should be used in computing v.

Form of Record. — This should provide for t, d, length of air column, number of segments, and s. These should be tabulated so that the average may be readily obtained for use in the computation of V.

III. Frequency of Vibration.

62. Problem. — *To measure the frequency of a tuning-fork.*

Sonometer Method. **Apparatus.** — A **sonometer** (Fig. 53), **meter rod**, **wire micrometer**, a **tuning fork**, and a **magnifier**. The tension device on the sonometer is such that the tension can be obtained in kilograms or pounds.

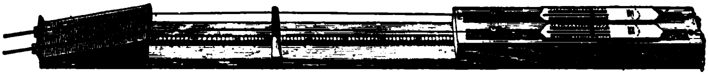

FIG. 53.

Directions.—When a wire is stretched between two points, it is shown in mathematical physics that $n = \frac{1}{2lr}\sqrt{\frac{T}{\pi d}}$, in which l and r are the length and radius in centimeters, respectively, d is the density, and T is the tension in dynes.

Stretch a wire on the sonometer, and by the movable bridge adjust the length till the wire sounds in unison with the fork. This tuning must be done with great care. Measure the length of the wire from center to center of rests, and also the diameter of the wire. This diameter should be measured in several places and the mean taken. Record the tension of the wire, using a magnifier to read the scale, so that a more accurate estimate of the fractional part of a scale division may be made. This tension must be reduced to dynes (Table XXII). The wire should be repeatedly adjusted to unison with the fork by moving the bridge, each new attempt being preceded by moving the bridge and destroying the previous adjustment. Each time that the wire is put in unison with the fork, the tension should be read and recorded. The density will probably be sufficiently accurate to take it from Table II of the Appendix. A more accurate method would be to cut out the piece of wire extending between the bridges, find its weight, and compute the density from the data. Substitute the mean values of l, r, d, and T in the formula $n = \frac{1}{2\,lr}\sqrt{\frac{T}{\pi d}}$, and the result will be the frequency of the fork.

Remarks. — The wire may be adjusted approximately to unison with the fork by placing a "rider" on the wire, and then, when the fork is vibrating, rest its stem on the sounding-board. If the "rider" is violently agitated or thrown off the wire, the unison is quite close. A "rider" is made by bending to a V-shape a strip of paper about 2 mm. wide and 4 cm. long. The final adjustment must be made by beats. In taking the tension be careful to observe the zero reading of the tension balance, if a balance be used, so that the true tension may be computed. The chief difficulty to be encountered will be that of tuning the wire to unison with the fork.

Graphic Method. **Apparatus.** — A **chronograph** (Fig. 54), a **shellac bath**, a **clock** provided with an electric contact, and an **induction coil** throwing a small spark. The chronograph is a brass cylinder mounted on an axis, one end of which has a screw thread on it, so that as the cylinder turns it also moves axially. The fork is fitted in a suitable holder, and a fine metal style is attached to the end of one prong.

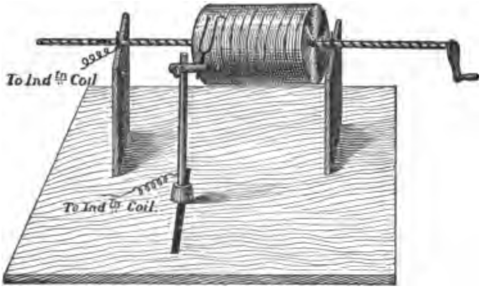

FIG. 54.

Directions. — A sheet of white calendered paper is wrapped about the cylinder and gummed at the edge. The paper is evenly smoked in a gas jet, or in the smoke of burning camphor gum. The paper should now be of an even gray color. The clock and the primary coil of the induction coil should be placed in circuit with two or three cells of a battery. One pole of the secondary circuit should be joined to the axis of the cylinder and the other to the fork. Adjust the fork so that the style rests gently on the paper with the plane of its prongs horizontal. Set the fork vibrating, close the circuit, and rotate the cylinder uniformly. The style should trace a wavy line and at every tick of the clock a spark should jump from the style to the cylinder, making a small dot on the smoked surface. Cut the paper from the cylinder by drawing the edge of a sharp knife along an element of the cylinder. Then holding the paper in a clip, dip it into a bath consisting of a dilute solution of shellac in alcohol and hang it up to dry. This will fix the wave forms on the paper so that the paper can be pasted in the notebook.

Count the number of waves between three successive dots. This divided by *two* will give the frequency of the fork. By taking dots as far apart as possible the error due to estimating fractions will be reduced. The number of seconds over which the count is spread should be *even* so as to eliminate any error in setting the mercury contact of the clock in the middle of the arc.

Siren Method. — **Apparatus.** — Cut out of cardboard a circular disk about 25 cm. in diameter. Draw on it a circle of 23 cm. in diameter and divide its circumference into fifty equal parts. At these points of division punch circular holes about 6 mm. in diameter. Mount this disk on the armature shaft of a Porter Electric Motor No. 2 (Fig. 55). This motor should have a speed indicator geared to the opposite end of the shaft. Use two storage cells or two Bunsen cells joined in series, to operate the motor. To regulate the speed roughly, place a rheostat in circuit; and for fine adjustment stretch a meter of No. 20 German silver wire on a board, connect one pole of the battery to one end of the wire, and the other to a spring contact that slides on this wire.

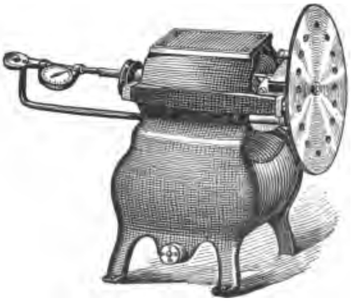

FIG. 55.

Directions. — Set the siren disk in motion, and, by means of a glass tube, direct a steady stream of air against the row of holes. By means of the rheostat and slide contact adjust the speed till the note due to the air puffs is of the same pitch as the fork. Observe the speed of the armature by marking the number of revolutions made in 20 sec. Multiply the speed per second by the number of holes in the siren disk; the product will be the frequency of the fork.

Remarks. — The chief difficulty will be to determine when the siren and fork are in unison, on account of the difference in the quality of the sounds. The experiment should be repeated several times. The first trials will doubtless be wide of the mark. With a little practice, the method will yield as good results as any that depends on the sense of hearing. The battery used must be strictly constant to insure uniform motion, and for like reason the pressure of the contact key must be constant.

Form of Record. — The student should prepare one.

63. Problem. — *To verify the laws of vibrating strings.*

Apparatus. — A **sonometer** (Fig. 53), a set of four **tuning forks** giving a major chord, and a **wire micrometer.** The sonometer should be provided with a means of attaching two wires, and measuring their tensions accurately. The simplest method of measuring the tension is by attaching the wire to the hook of a spring balance, and the most satisfactory device for regulating the tension is a rod across the head of the instrument provided with a hook at one end for attaching the wire and a winged nut at the other.

Directions. — *Law of Length.* — Stretch a fine steel wire on the sonometer, giving it sufficient tension to yield a clear tone, and then by the movable bridge adjust the length till it sounds in unison with the fork of lowest frequency. Measure the length of the wire with a meter rod.

Repeat this operation several times and find the mean of the lengths. In like manner find the length of wire that vibrates in unison with each of the other forks. The frequencies of the forks forming a major chord are as 4 to 5 to 6 to 8. Multiply each length of wire by the corresponding frequency, and the products should be constant to prove that the number of vibrations varies inversely as the length of the wire.

Law of Diameter. — Stretch two steel wires of different diameters, giving them the same tension. The larger wire should have the movable bridge under it before setting the tension. Shorten the larger wire by moving the bridge till the two wires are exactly in unison, and measure the lengths of both. This should be repeated several times. Measure with the wire micrometer the diameter of each wire in several places. Multiply the length of each wire by the corresponding mean diameter. If these products are constant, then the number of vibrations varies inversely as the diameter.

Law of Tension. — Stretch two steel wires of the same diameter, giving them different tensions. The movable bridge must be under the wire of lowest tension before noting the tension. Move the movable bridge till the wires are in unison, and measure the length of the wires. Record the lengths of both wires and the corresponding tensions. Divide each length by the square root of the corresponding tension. If these quotients are equal, it will show that the number of vibrations varies as the square root of the tensions.

Law of Density. — Stretch a brass and a steel wire, giving them the same tension. The one of lower pitch should have the movable bridge under it before making the final adjustment of tension. By means of the bridge bring the two wires to unison, and measure their lengths. Repeat this several times. Measure the diameters of the wires in several places. Multiply the length of the brass wire by the ratio of its diameter to that of the steel. The length obtained by this operation will be that of a brass wire in unison with the steel whose diameter is the same as that of the steel. (Why?) Find the density of each wire from Table II of the Appendix. Multiply the length of each wire by the square root of the corresponding density. The corrected length must be used. Constant products would imply that the number of vibrations varies inversely as the square root of the density.

Remarks. — The zero reading of each tension balance should be recorded, and each tension must be corrected accordingly. A magnifier should be used in reading tensions. In case that the sonometer has tuning posts for adjusting the tension, be careful not to screw the tuning posts too far into the headpiece. Every time a change is made in the wires, the posts should be turned back to their positions at the beginning.

Form of Record. — Enter results as follows: —

Zero reading of bal. $A + 0.5$ kgm.; of bal. $B - 0.2$ kgm.
Forks used C, E, G, C'. Frequency ratios, 4, 5, 6, 8.

		Lengths of Wires			
Law I		49.5 cm. 49.1 " 49.6 " 49.4 "	39.0 cm. 39.3 " 39.6 " 39.4 "	33.0 cm. 32.9 " 32.8 " 33.0 "	24.8 cm. 24.9 " 24.9 " 25.0 "
	Mean	49.4 cm.	39.3 cm.	32.9 cm.	24.9 cm.
	Frequency ratios	4	5	6	8
	$n \times l$	197.6	196.5	197.4	199.2

		Diameters of Wires		Lengths of Wires	
		Small	Large	Small	Large
Law II		0.43 mm.	0.88 mm.	81 cm.	39.6 cm.
		0.43 “	0.89 “	81 “	39.8 “
		0.43 “	0.88 “	81 “	39.7 “
		0.44 “	0.89 “	——	——
		0.44 “	0.88 “	——	——
	Mean	0.43 mm.	0.88 mm.	81 cm.	39.7 cm.
	$d \times l$	34.83	34.94	——	——

		Bal. *A*	Bal. *B*	Length of Wire *A*	Length of Wire *B*
Law III	Observed tension	11.8 kgm.	7.0 kgm.	81 cm.	64.8 cm.
	Correct'd tension	11.3 “	7.25 “	81 “	64.8 “
	Sq. rt. of tension	3.36	2.69	81 “	65.0 “
	$l \div \sqrt{t}$ . . .	24.1	24.1	81 cm.	64.87 cm.

		Diameters		Lengths	
		Steel	Brass	Steel	Brass
Law IV		0.432 mm.	0.645 mm.	81 cm.	51.8 cm.
		0.433 “	0.648 “	81 “	51.9 “
		0.430 “	0.644 “	81 “	51.9 “
	Mean	0.432 mm.	0.646 mm.	81 cm.	51.9 cm.
	Density, or D .	7.816	8.440		
	$\sqrt{D}$	2.796	2.905		
	Length corrected for d . . .	81 cm.	77.61 cm.		
	$l \times \sqrt{D}$ . . .	226.5	225.5		

CHAPTER VI.

LIGHT.

I. Images through Small Apertures.

64. Problem. — *To ascertain how images are formed through small apertures, and what conditions govern their size, brightness, and definition.*

Apparatus. — A **lamp**, two **screens**, and a sheet of thin cardboard. The screens are made of thin pine board, each one being about 30 cm. square and fitted to a base to support it vertically. One of them has an aperture at its center 5 cm. square. The sheet of cardboard is rolled into a cylinder of sufficient diameter to encircle the lamp, and a circular aperture 2 mm. in diameter is cut in it opposite the lowest point of the lamp flame, as it encircles the lamp.

Directions. — Tack a piece of paper over the aperture in the screen. Cut a hole about 3 mm. square in this paper. Place in line the lamp, the screen with aperture, and the screen without aperture, in the order named, and about 15 cm. apart. Place the paper cylinder around the lamp so that the light, passing through the aperture of the cylinder, is incident on the first screen and forms on it an inverted image of the lamp flame. Now adjust the apparatus so that some of the light passes through the first screen to the second one. By observing what part of the image is over the aperture in the first screen, it will be known from what part of the flame the light comes that produces the illuminated figure on the second screen. Trace with a pencil the outline of this figure, being careful not to move the screen. Of what is it a picture? How is it formed? Adjust the cylinder about the lamp, so as to take light successively from a series of points in the border of the flame, tracing each time the figure that is formed on the screen. Pass a pencil line through the approximate centers of these figures. Now remove the cylinder, observe the image obtained, and compare it with the pencil sketch. What is the image given by the small aperture shown to be?

Tack a piece of paper over the aperture of the first screen, having cut three small apertures in this paper, one square, one round, and one triangular. These apertures should be of approximately equal areas. Compare the images formed, noting relative size, relative brightness, and relative definition.

Try apertures of various sizes, as 2 mm. square, 4 mm. square, 8 mm. square, 16 mm. square, 32 mm. square, keeping the distances between lamp, aperture, and screen constant. Ascertain when the image loses all resemblance to the object. In this last case determine whether the image can be restored by increasing the distances between lamp, aperture, and screen.

Use a small aperture and vary the distance between lamp, aperture, and screen as follows: Lamp and screen each 10 cm. from aperture; lamp 10 cm. and screen 20 cm.; lamp 20 cm. and screen 20 cm.; lamp 20 cm. and screen 10 cm.; lamp 30 cm. and screen 10 cm.; lamp 10 cm. and screen 30 cm.; both 30 cm. What law governs the dimensions of the image?

Remarks. — The experiment must be conducted in a darkened room. In describing images note the *position*, whether erect or inverted; *size*, whether larger or smaller than the object, as shown by length or breadth; *brightness; definition*, that is, the clearness of outline. The discussion should consider each case of the problem, pointing out clearly what has been determined. The images will be brighter if a piece of white paper is tacked on the receiving screen. The figure obtained in the first case should be preserved and made a part of the record.

Form of Record. — The following form may be used:—

CONDITIONS	OBSERVATIONS
Write each condition of the experiment as described in the directions.	Write opposite each set of conditions a description of the image obtained, recording measurements when any are made.

65. Problem. — *To ascertain why an image of every object is not seen impressed on every other object in accordance with the principle governing the formation of images through small apertures, any point in space being considered as such an aperture.*

Apparatus. — Same as in Art. 64.

Directions. — Tack a piece of paper over the aperture in the screen, and place the lamp and the two screens as in Art. 64. Make a hole through the paper with a heavy darning needle. An image will be formed on the second screen. Make other holes in the paper and observe the positions of the images. Fill the paper full of holes and observe the effect. Infer the effect if the screen were all holes. Try it. What do the results show the answer to the problem to be?

Remarks. — This experiment must be conducted in a darkened room. The screen must not be too far from the aperture, as the images will not be sufficiently bright to insure success.

Form of Record. — Enter the results after the manner of Art. 64.

II. Photometry.

66. Problem. — *To measure the candle power of a kerosene lamp, or gas jet, or incandescent lamp.*

Apparatus. — A **Joly photometer** and a **sperm** or **paraffin candle.** A simple photometer bench may be constructed as follows: Fasten two meter rods end to end on a board. Three grooved blocks, *A*, *B*, and *C*, slide on these rods; one block carries the lamp or light to be measured, the second the photometer, and the third the standard candle (Fig. 56). Both the light to be measured and the standard candle should be inclosed in wooden or pasteboard boxes, blackened on the inside. Each box should have an opening in the top, holes near the bottom for draught, and an opening in the side opposite the light. The Joly photometer consists of two rectangular blocks of paraffin, each about 1 cm. thick and 5 cm. square. The white crystalline variety of paraffin should be used. These cakes are separated by a sheet of tinfoil or paper, and mounted across

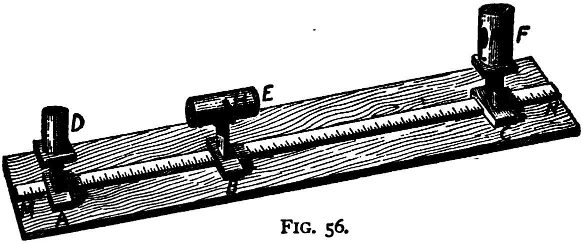

FIG. 56.

the middle of the horizontal pasteboard cylinder, *E*. This cylinder is about 20 cm. long, open at the ends, and with an opening at the middle, *H*, exposing the edges of the paraffin block. It is mounted on the sliding block at the middle of the photometer bench, and at such an elevation that its axis is in line with the light to be measured and the standard candle. The candle is of the size known as "sixes," and when lighted, the wick should be kept trimmed so as to maintain a flame as near 3 cm. long as possible.

With a photometer constructed in this manner the work is not materially affected by other lights in the room, and it is possible to operate several such instruments in the room without interference. The light incident on the paraffin by diffusion through the cake illuminates the edges, and when both sides are equally illuminated the edges are equally bright.

Directions. — Place the photometer cylinder so that the middle of the paraffin block is at the common zero of the linear scales. Place the light to be measured on one end of the bench. Adjust its height to the axis of the photometer cylinder, and then place over it the cylindrical box designed to screen its light from the eye as well as from the other workers in the room. Place the candle on the bench and on the opposite side from the light to be measured. Trim the flame to the required height and cover the candle with its screen. Now move the candle to and fro along the bench till the edges of the paraffin blocks are equally bright. Read from the linear scales the distances of the two lights from the photometer screen. The ratio of the squares of the distances of the light and candle from the photometer will be the candle power. Repeat with the light at different distances from the screen. Make part of the measurements with the flat side of the flame toward the photometer screen and part with the edge. Average separately. Why is there a difference in candle power?

Remarks. — It will be noticed in adjusting the distance of the candle that it may be moved a short distance without the eye being able to detect any change in the relative brightness of the edges of the paraffin block. Two points should be found at which the change is first detected and takes place in opposite directions. The mean of the distances of these points from the photometer may be considered as the true distance for equal illumination. In describing the light that is the subject of measurement, the specifications should be complete. For example, in the case of a lamp burning kerosene, the name of the burner, the size, the kind of oil used, etc., should be recorded. The experiment should be conducted in a darkened room.

Form of Record. — This should provide for the distances, their squares, and the ratios of these squares. The candle power will be the mean of these ratios.

III. Reflection of Light.

67. Problem. — *To verify the law of reflection of light.*

Apparatus. — A piece of **plane mirror** 8 cm. by 10 cm., fastened by rubber bands to a rectangular wooden block, a horn **protractor**, and three pins.

Directions. — Tack a sheet of printing paper on the table. Draw a straight line, *MN*, upon it (Fig. 57). At a point, *O*, in the line, erect a perpendicular, *OP*. This may be accurately done by the aid of

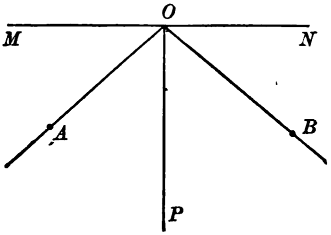

Fig. 57.

a protractor. Draw the oblique line, OA, and set a pin, A, in this line and normal to the paper and a second pin at O. Now place the mirror on the line, MN, with its plane normal to the paper. Set a third pin at B, so that on sighting along BO the image of A is seen directly behind O. Remove the mirror and draw the line BO with a finely pointed pencil. Measure the angles, BOP and AOP (26). Repeat several times, giving the angle of incidence, AOP, different values. Calculate the error of each observation.

Remarks. — The alignment should be done close to the paper to reduce errors due to the pins not being set normally to the surface of the paper.

Form of Record. — The student to devise one.

68. Problem. — *To measure the angle of a triangular glass prism.*

Apparatus. — A triangular **glass prism**, preferably about 25 mm. face and 5 cm. long, three **straightedges**, each about 25 mm. wide and 30 cm. long, and a **protractor.**

Directions. — The polished faces of the prism will act as mirrors, and the edge, EH, of the straightedge, HF (Fig. 58), can be seen reflected in AB by looking along the line, NO, and a similar thing is true of the other edge. AB may be considered as the surface of a plane

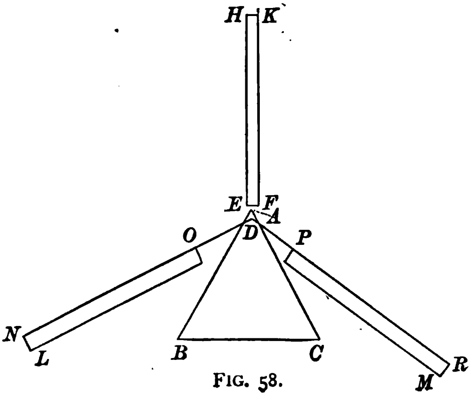

FIG. 58.

mirror that generates the angle of the prism, *BAC*, in revolving on *A* as a pivot from the position, *AB*, to the position, *AC*; and the angle, *NDR*, may be considered as the angle through which the reflected ray rotates during this displacement of the mirror. Now it is shown in elementary physics that the displacement of the reflected ray caused by a rotating mirror is twice the angle through which the mirror turns. Hence, one half of the angle, *NDR*, is equal to the angle of the prism.

Tack a sheet of paper on a thin board about 30 cm. by 60 cm., stand the prism on end near the center of the board, and place one of the straightedges, *HF*, as shown in the figure. Now rotate the board till the light from the window brightly illuminates the edge, *HE*, and place a second straightedge so that its edge, *NO*, is exactly in line with the reflection of *HE* seen in the face, *AB*, of the prism. Again rotate the board so that *KF* is illuminated, and place the ruler, *MP*, so that its edge, *RP*, is in line with the reflection of *FK*. Draw the lines, *ND* and *RD*, with a finely pointed pencil along the edges of the rulers. Produce these lines till they intersect and measure the angle formed, using the protractor for the purpose. Repeat this several times and obtain the mean of the trials. Half of this mean will be the angle of the prism.

Remarks. — The axis of the prism should be normal to the paper, and the alignment of the rulers should be done with the greatest care. Pins or darning needles may be used instead of rulers. By conducting the experiment in a dark room, and having a beam of light incident on the edge of the prism, it will be divided, part being reflected by the face, *AB*, and part by the face, *AC*. Set needles, as at *O* and *P*, in these reflected beams. These will cast shadows on the paper. Then set pins, as at *N* and *R*, exactly in the centers of these shadows. Straight lines through *N* and *O* and through *R* and *P* will form an angle double that of the prism.

Form of Record. — Provision should be made for a succession of values obtained for the angle at *D*. Half of the mean of these values will be the angle at *A*.

IV. Mirrors.

69. Problem. — *To prove that the image of an object in a plane mirror is situated as far back of the mirror as the object is in front.*

Apparatus. — Same as in Art. 67.

Directions. — Tack a sheet of printing paper on the table. Draw a straight line upon it, and place the mirror in the normal plane containing this line. Set a pin, N_1 (Fig. 59), normally to the paper and about 5 cm. in front of the mirror, *MR*. Set a second one, N_2, distant at least 5 cm. from N_1, in exact alignment with N_1 and its image, *I*, in the mirror. To one side of these set two more pins, N_3 and N_4, so that the line of sight marked by them passes exactly through *I*, the image of N_1. Now remove the mirror and with a finely pointed pencil draw straight lines through the points marked by the pins, producing these lines until they intersect. The point of intersection, *I*, is the position of the image of N_1. Measure the distance of *I* and N_1 from *MR*, recording the former as the distance of the image, and the latter as the distance of the object from

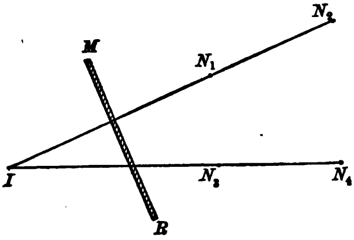

FIG. 59.

the mirror. Make a large number of trials, varying the positions of the pins. Calculate the error of each trial. Vary the inclination of the line of sight, N_4N_3, and note the effect on the results. Explain. Try mirrors of different thicknesses, comparing the errors. Explain.

Remarks. — The farther apart the pins are placed the more accurately they mark out a line. Why? What sources of error must be guarded against?

Form of Record. — The student should devise one.

70. Problem. — *To measure the focal distance of a concave spherical mirror.*

Apparatus. — A **concave mirror**, a **meter rod**, a **lamp**, and two wooden or cardboard **screens**. An optical bench such as is shown in Fig. 56 is convenient for this problem. In the absence of it fasten the meter rod lengthwise of the middle of a board that is about 20 cm. wide. The screens, each about 20 cm. high by 15 cm. wide, are supported vertically by being tacked each to a rectangular piece of board. They are placed on opposite sides of the meter rod with their planes at right angles thereto. In one screen there is a circular aperture 2.5 cm. in diameter and two threads are cemented across its center at right angles. The mirror is supported with its vertex exactly over the zero of the scale, and with the reflecting surface toward the screens. The center of the aperture and the vertex of the mirror should be in a horizontal line.

Directions. — *First Method.* Place a lighted lamp back of the apertured screen to illuminate the cross threads. Slide the other screen along the scale till a clear and well-defined image of the cross threads is formed upon it by the mirror. Record the distances of the screens from the mirror. If these distances are represented by *a* and *b*, then the focal length of the mirror can be found by substituting in the formula $f = \frac{ab}{a+b}$. Make several trials, varying the position of the apertured screen.

The formula, $f = \frac{ab}{a+b}$, is derived as follows: Let *MN* (Fig. 60) represent a concave spherical mirror, *C* its center of curvature, *CD* its radius of curvature and represented by *r*, *A* an object whose distance, *AD*, from the mirror is *a*, *H* the image of that object and its distance from the mirror is *b*, and *FD* the principal focal distance, represented by *f*. Since *CB* bisects the angle *ABH*, then $\frac{AB}{BH} = \frac{AC}{CH}$. But, in spherical mirrors, *BD* is small in comparison with the radius of the mirror. Hence, *AB* may be considered as equal to *AD* without sensible error, and in like manner $BH = HD$. Make these substitutions, and we have $\frac{AD}{HD} = \frac{AC}{CH}$. But $AD = a$, $HD = b$, $AC = AD - CD = a - r$, and $CH = CD - HD = r - b$. Therefore, $\frac{a}{b} = \frac{a-r}{r-b}$, from which $r = \frac{2ab}{a+b}$ and $f = \frac{r}{2} = \frac{ab}{a+b}$.

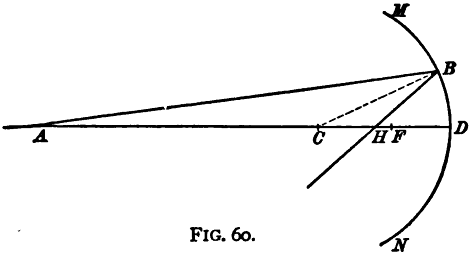

FIG. 60.

Second Method. In the formula, $f = \frac{ab}{a+b}$, if $b = a$, then $f = \frac{a}{2}$, that is, if the object in front of a concave spherical mirror is so placed that the image and object coincide, then the focal distance of the mirror is one half of the distance of the object from the mirror, in other words, the object marks the center of curvature of the mirror. Support the mirror as in the first method, and in place of the screens take a square wooden block, in which is stuck a darning needle, with a triangular piece of stiff white paper inserted in the eye. The point of this paper should be in the principal axis of the mirror. Place this block at some distance in front of the mirror. With the eye in line with the paper index and the vertex of the mirror, and a meter or so farther from the mirror than the needle, move the block and incline the mirror, if necessary, till an inverted image of the needle and index is seen in the air directly over the real needle. Now adjust the apparatus till the point of the paper seems to coincide with the image of that point. Read from the scale the distance of the paper index from the vertex of the mirror. Half of this distance will be the focal length. Make several independent trials.

Remarks. — The first method succeeds best in a darkened room, but for the second method good light is necessary.

Form of Record. — The results may be tabulated as follows: —

First Method			Second Method	
a	*b*	*f*	*r*	*f*
—— cm.	—— cm.	—— cm.	—— cm.	—— cm.
—— cm.	—— cm.	—— cm.	—— cm.	—— cm.
etc.	etc.	etc.	etc.	etc.
Mean		—— cm.		—— cm.

71. Problem. — *To measure the focal distance of a convex spherical mirror.*

Apparatus. — A **convex mirror**, a **linear measure**, a sheet of cardboard, in which is a circular aperture 3 cm. in diameter, with two cross threads at right angles cemented across its center, and a strong beam of light, either sunlight or an electric arc light.[1]

Directions. — If MN (Fig. 61) represent a convex spherical mirror whose vertex is K, and $GABH$ is a beam of light incident upon it through the aperture, GH, in the screen; this beam will be reflected as a pencil of light, $DABE$, diverging from F, the principal focus. Let $GH = AB = a$, $DE = b$, $LK = c$, and $KF = f = \frac{1}{2}r$. By similar triangles $\frac{AB}{DE} = \frac{KF}{LF}$. Hence, $\frac{a}{b} = \frac{\frac{1}{2}r}{c + \frac{1}{2}r} = \frac{r}{2c + r}$, from which $r = \frac{2ac}{b - a}$ and $f = \frac{ac}{b - a}$. Values for a, b, and c can be obtained in the following manner: Support the screen so that the light is normal to it, some of it passing through the aperture and striking the mirror, which is placed parallel to the screen, and perhaps 15 to 25 cm. distant from it. When properly adjusted, there should be a circular spot of light at the

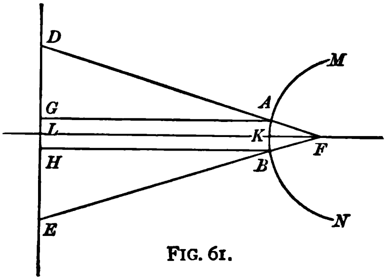

FIG. 61.

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[1] To use sunlight for experimental purposes, a porte lumière (143) must be placed in a south window to reflect the light into the room. The objections to sunlight are that it is very uncertain, and but two or three students can be accommodated at a time. The best light for laboratory purposes is obtained in the manner described in Art. 4.

vertex of the mirror, and a circular spot of light, due to reflection, on the screen and concentric with the aperture. Measure with dividers and scale the diameter of the aperture, GH or a, along one of the cross threads; the diameter of the reflected spot, DE or b, on the screen, along the shadow of the same cross thread; also the distance, LK or c, of the screen from the mirror. Substitute these values in the formula $f = \frac{ac}{b-a}$. The results should be tabulated and the mean determination obtained.

Remarks. — The principal difficulty in this problem is in the uncertainty of the boundary of the reflected spot of light, caused by the spherical aberration of the mirror and by multiple reflection. The definition is improved by placing the mirror several meters from the electric light.

Form of Record. — The student is to devise one.

72. Problem. — *To ascertain how a concave spherical mirror disposes of a pencil of light.*

Apparatus. — A **concave spherical mirror** of known focal length, a **convex lens** of about 50 cm. focus and 8 cm. diameter, a **meter rod**, a **screen**, a stout wire 20 cm. long set normally in a wooden block, and a strong beam of light.

Directions. — Support the lens so that the beam of light is incident upon it. The dust floating in the room will show that the beam issues from the lens as a converging pencil, passing approximately through a point, the focus of the lens, and continues as a diverging pencil from that point. Place the wire to mark the focus. This point may be considered as the radiant point of a diverging pencil. Set the mirror so that this pencil is incident upon it, and then by moving the mirror toward or away from the wire, as may be necessary, give the radiant point the following positions relative to the characteristic points of the mirror: —

First. — Place the mirror so that this radiant point is distant from the mirror farther than the center of curvature, and find a position of the screen such that there is a sharp image of the wire upon it. The screen will then mark the point that is a conjugate focus to the radiant point. Measure the distances of these points from the mirror, and record. Follow this procedure in each case to be examined.

Second. — Place the mirror so that the radiant point is at the center of curvature, and as before find the conjugate focus. What is the inference?

Third. — Place the mirror at a distance intermediate between the radius of curvature and the focal distance. To which case is this one closely related?

Fourth. — Place the mirror at its focal distance from the radiant point. What defect in the mirror is made apparent?

Fifth. — Place the mirror at a distance less than the focal length from the radiant point.

Sixth. — Place the mirror so that the converging pencil is incident upon it. Can the image of the wire in every case be obtained on the screen?

Remarks. — It will be necessary to turn the mirror a little to one side, so that the reflected light may be received on the screen without the screen's cutting off the light from the mirror. In discussing this problem, the student should show by drawing geometrical figures that these several cases should arise, and then, comparing the results, point out whether he has verified these conclusions. The electric lantern (4) is well suited to the demands of this problem. The experiment must be conducted in a darkened room.

Form of Record. — The following example illustrates the manner of tabulating the results :—

POSITION OF RADIANT POINT	CHARACTER OF PENCIL	EFFECT OF MIRROR	DISTANCE FROM RADIANT POINT TO MIRROR	DISTANCE FROM MIRROR TO CONJ. FOCUS	POSITION OF CONJ. FOCUS
Inf. dist.	Parallel	Converge	Infinite	32 cm.	At Prin. focus
Beyond center	Diverging	Converge	70 cm.	59 "	Bet. *F* and *C*
At *C*	Diverging	Converge	64 "	64 "	At *C*
Bet. *F* and *C*	Diverging	Converge	46 "	98 "	Beyond *C*
etc.	etc.	etc.	etc.	etc.	etc.

73. Problem. — *To determine the character of the images of an object given by spherical mirrors.*

Apparatus. — A **convex mirror**, a **concave mirror**, each of known focal length, a **lamp**, **meter rod**, and a white **screen.**

Directions. — Place in line on a table in a darkened room, the lamp, screen, and concave mirror, in the order named. Adjust the height of the mirror so that its vertex is in a horizontal plane passing through the flame of the lamp. Give the mirror successively the positions described in Art. 72, and locate the image of the lamp whenever possible, by receiving it on the screen. Note the characteristics of each image, such as size, position, kind, brightness, definition.

In like manner study the convex mirror. What effect does the distance of the object from the convex mirror have upon the image? Compare the brightness and the definition of the image given by the concave mirror when the light is cut off by a paper ring from all but the central portion of the mirror, with that when the whole surface of the mirror is used. Explain.

Remarks. — In discussing this problem, show, by drawing accurate figures, that there are several distinct cases of images, and then compare the results with theory.

Form of Record. — The following example illustrates the manner of tabulating the results: —

KIND OF MIRROR	FOCAL LENGTH	POSITION OF OBJECT	POSITION OF IMAGE	CHARACTER OF IMAGE
Concave	32 cm.	70 cm.	59 cm.	Real, inverted, smaller than object.
Concave	32 "	64 "	64 "	Real, inverted, same size as object.
etc.	etc.	etc.	etc.	etc. etc.

V. Refraction of Light.

74. Problem. — *To measure the index of refraction of water.*

Apparatus. — Cement on one face of a glass rectangular battery jar a circular paper protractor, 10 cm. in diameter, from which the center has been removed. Across this circular opening cement two threads, marking the horizontal and vertical diameters respectively. The position of the protractor should be such that the horizontal thread passes through the zeroes of the scale (Fig. 62). In a rectangular strip

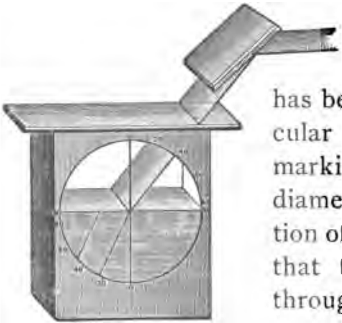

FIG. 62.

of cardboard somewhat wider and longer than the top of the jar, cut crosswise a slit 2 mm. wide and 5 cm. long. The remaining faces of the jar should have a coat of black lacquer.

Directions. — Fill the jar with water exactly to the horizontal thread. Cover the top of the jar with the slotted cardboard. Set the apparatus in the sunlight; with a plane mirror reflect the light through the slit, adjusting the slit so that the ribbon of light passing through it strikes the water exactly back of the center of the circular opening in the face. If sunlight is reflected into the room by a porte lumière, or if an electric lantern is used, then with a plane mirror reflect the light into the tank as shown in Fig. 62. Read off on the protractor scale the angle of incidence and that of refraction, taking the readings to the center of the edge of the band of light. From Table XIX of the Appendix find the sines of these angles and divide the former by the latter. The quotient will be the index of refraction. By moving the slit and changing the position of the mirror, variations may be made in the angles. Several trials should be made and the arithmetical mean calculated.

Remarks. — In reading the angles, the line of sight must be normal to the scale. Any transparent liquid might be substituted for water and its index of refraction found in the same way.

Form of Record. — The student should devise one. It should show not only the angles, but the sines of the angles as well.

75. Problem. — *To measure the index of refraction of glass.*

Apparatus. — A **triangular glass prism** with ends squarely cut off, a **protractor**, a **board** about 30 cm. square, with one side covered with white paper, and two flat **rulers**.

Directions. — Measure the angle, *A*, of the prism as directed in Art. 68. Then place the ruler, *DE*, on the paper with its edge making an acute angle with the face, *AB*, of the prism (Fig. 63). Rotate the board so that the light strikes the edge of this ruler, and then place the second ruler, *FG*, so that by sighting along its edge it seems to be in line with the edge, *DE*, seen through the prism. The edge, *DE*, of the ruler seen through the prism will be colored with the spectrum tints. The alignment should be made with reference to the green color. Now turn the prism on its axis till the edges of the rulers are as nearly equally inclined to the faces of the prism as the eye can determine. This may put the rulers a little out of alignment, if so, a readjustment will be necessary. Endeavor to secure a perfect alignment of the rulers and at the same time have them equally inclined to the faces of the prism. With a finely pointed pencil draw lines along these rulers producing one of the lines to *K*. These lines will represent, respectively, an entering ray and an issuing ray of light. The angle *KHG* is the angle of least deviation, since *DE* and *GF* are equally inclined to the faces of the angle *A*. Measure the angle *KHG* with the protractor. Repeat the work several times, and find the mean. If the angle of the prism be represented by *a* and the deviation by *d*, then the index of refraction will

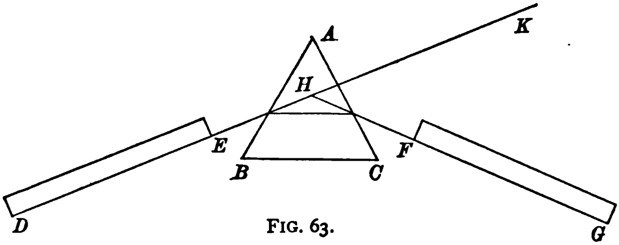

FIG. 63.

be found by substituting in the formula $i = \frac{\sin \frac{1}{2}(a + d)}{\sin \frac{1}{2} a}$. In applying this formula, find half of the sum of a and d, with this value enter Table XIX of the Appendix and obtain the sine. Divide this by the sine of half of the angle a. The derivation of this formula is found in works on Geometrical Optics.

Remarks. — Each sample of glass has its characteristic index of refraction. Hence, the error of the work can not be determined, unless the index of the sample has been carefully measured with a spectrometer in the hands of an expert.

Form of Record. — This should provide for the measurements of the angles, BAC (68) and KHG, as well as for the sines of $\frac{1}{2}(a + d)$ and $\frac{1}{2} a$.

VI. Lenses.

76. Problem. — *To measure the focal distance of a convex lens.*

Apparatus. — A **convex lens,** the **screens,** and **optical bench** of Art. 66, a **lamp,** and a small **telescope** or **spyglass.** In the absence of an optical bench, fasten two meter rods in a straight line on a board with their zeroes in common.

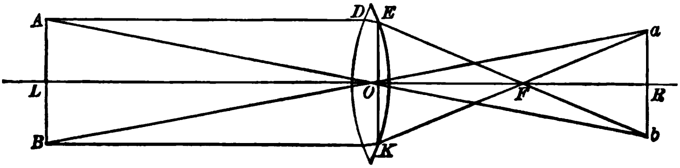

FIG. 64.

Directions. — *The Image Method.* Let a (Fig. 64) represent the distance of an object, AB, from the convex lens, *DK*; b the distance of the image, ab; and f the focal distance, *OF*. The similar triangles, *AOB* and *aOb*, give the proportion $\frac{AB}{ab}=\frac{LO}{OR}$, and the similar triangles *EFK* and *aFb* give $\frac{EK}{ab}=\frac{OF}{FR}$. But in thin lenses *EK* may be considered, without sensible error, to be equal to *AB* and to pass through *O*. Hence, $\frac{LO}{OR}=\frac{OF}{FR}$, that is, $\frac{a}{b}=\frac{f}{b-f}$, from which $f=\frac{ab}{a+b}$. To determine the quantities a and b, support the lens between the two screens in a vertical plane, passing through the zero of the scale on the optical bench. Place the lamp behind the apertured screen to illuminate the cross threads. The aperture and optical center of the lens should be approximately in a horizontal line. Now find by trial a position for these screens, so that a sharply defined image of the cross threads of the one is formed on the other. The scale of the optical bench will give the distances, a and b, of these screens from the lens. Compute f from the formula, $f=\frac{ab}{a+b}$. Repeat the work for different positions of the apertured screen. The mean of the several values obtained is the focal length of the lens.

The Telescope Method. If a telescope is accurately focused on some clearly marked object, as the side of a house a mile or more distant, it may be said to be adjusted to view parallel rays of light, since rays coming to the eye from a point that far distant are sensibly parallel. If rays diverge from the focus of a convex lens, and pass through the lens, they issue parallel. Hence, a telescope adjusted to parallel rays is adapted to view light that comes to it through a convex lens if it emanates from the focus of that lens. These considerations suggest the following: Focus the telescope on some distant object. Now place the lens in front of the telescope, and look through the telescope and lens at a printed card fastened to a vertical screen. Find by trial a position for the screen at which the words are most clearly seen. Measure the distance of this screen from the lens. This will be the focal distance sought. Obtain the mean of several trials and compare with that obtained by the first method.

Form of Record. — The student is to devise one.

77. Problem. — *To measure the focal distance of a concave lens.*

Apparatus. — A **concave lens**, also the apparatus of Art. 71.

Directions. — If a beam of light be incident on a concave lens, it will issue as a pencil diverging from the principal focus. Let *SC*, *S'D* (Fig. 65), be a beam of light incident on the lens, *MN*. Let *AB* be the intersection of the diverging pencil by a plane normal to the beam. Represent the distance *AB* by *a*, *CD* by *b*, and *EH* by *c*. *CD* may be considered, without sensible error, as a straight

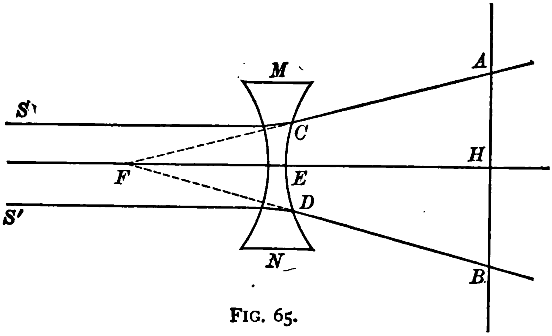

FIG. 65.

line parallel to AB. Then the similar triangles, FCD and FAB, give the proportion $\frac{FH}{FE}=\frac{AB}{CD}$, from which

$$\frac{f+c}{f}=\frac{a}{b} \text{ and } f=\frac{bc}{a-b}.$$

Cover one face of the lens with a disk of paper in the center of which is a circular opening 2 cm. in diameter. Cement a thread across the opening, marking a diameter. Support the lens in a vertical plane with the uncovered face toward the beam of light. Intercept the diverging pencil issuing from the lens with a screen placed parallel to the lens. The distance of the screen may be between 20 and 50 cm. Measure with dividers and scale the diameter of the aperture along the thread, the diameter of the spot of light on the screen as marked out by the shadow of this thread, and the distance of the screen from the lens. This will give a, b, and c respectively. Substitute these values in the formula $f=\frac{bc}{a-b}$. Make several trials, varying the distance of the screen from the lens.

Remarks. — The chief difficulty will be found in measuring accurately the diameter of the spot of light on account of spherical aberration causing such a blurring of the edges.

Form of Record. — The student to devise one.

78. Problem. — *To ascertain how a lens disposes of a pencil of light.*

Apparatus. — A **convex lens** and a **concave lens**, each of known focal length, and the apparatus of Art. 72.

Directions. — Proceed as in Art. 72, except that the convex lens to be tested is substituted for the mirror and is placed between the radiant point and the screen. Try the following conditions: —

First. With the radiant point distant from the lens more than twice the focal length.

Second. With the radiant point at twice the focal distance.

Third. With the radiant point at less than twice and more than once the focal distance.

Fourth. With the radiant point at the focus.

Fifth. With the radiant point at less than the focal distance.

Sixth. Cover the center of the lens with a disk of paper and locate the distance of the conjugate focus from the lens. Then cover all but the central part of the lens and locate the conjugate focus. Explain.

Seventh. Let a converging pencil be incident on the lens and ascertain whether the general effect is distinctively changed by changes in the distance of the lens from the radiant point.

In all these cases record the distance of the radiant point and its conjugate focus from the lens.

Substitute the concave lens for the convex one and try the following cases: —

First. Ascertain whether the distance of the radiant point from the lens, when the pencil is diverging, affects the general character of the results.

Second. Using the converging pencil, place the lens so that its focus is beyond the point to which the pencil converges.

Third. As in the last case, except that the focus coincides with the point to which the pencil converges.

Fourth. As in the last case, except that the focus is between the lens and the point of convergence of the pencil.

Remarks. — The discussion of this problem should follow quite closely that suggested in Art. 72.

Form of Record. — Consult that of Art. 72 and adapt it to this problem.

79. Problem. — *To ascertain the character of the images formed by lenses.*

Apparatus. — A **convex lens** and a **concave lens**, each of known focal length, an **optical bench**, a **lamp**, and a white **screen**. Figure 66 suggests a simple optical bench with supports.

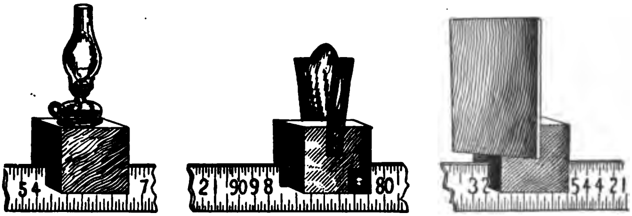

FIG. 66.

Directions. — Mount the convex lens in the vertical plane through the zero of the optical bench scale. Place the lamp, convex lens, and screen on the optical bench in the order named. Give the lamp successively the following positions, and in each case locate the image, using the screen whenever the image is such as can be shown in that manner.

First. At more than twice the focal length of the lens.

Second. At twice the focal length.

Third. At less than twice and more than once.

Fourth. At the focus.

Fifth. Between the focus and the lens.

In each case record the distance of the image and the object from the lens and also the position, size, and kind of image.

Compare the definition of the image formed when the whole lens is used with that given when the outer part of the lens is covered with a ring of paper. Explain.

Substitute a concave lens for the convex, ascertain the character of the image, and determine the effect of varying the distance of the object from the lens.

Form of Record. — Enter the facts and discuss them after the manner suggested in Art. 73.

80. Problem. — *To measure the magnifying power of a lens or telescope.*

Apparatus. — Two **linear scales**, exactly alike, and divided to fifths of millimeters, **clamps** for supporting them, a **convex lens**, and a **telescope**.

Directions. — Mount one of the linear scales in a vertical position, then support the lens in a clamp and find by trial a position giving a clear image of the scale when viewed through the lens. Place the second scale nearly in line with the lens and the first scale, and at about 25 cm., the distance of distinct vision, from the eye when looking through the lens. Now look with one eye through the lens at the first scale, and with the other eye at the second scale but not through the lens. When the eye becomes accustomed to the conditions of the experiment, the magnified image of one scale will be seen superposed on the other scale. Count how many divisions of the magnified scale exactly cover a certain number on the other. Divide the latter by the former, the quotient will be the magnifying power. Make several trials and find the mean. Compare the result with that obtained by the formula $1 + \frac{25 \text{ cm.}}{f}$, where f is the focal distance of the lens.

The magnifying power of a telescope is measured in a similar manner. A scale of sufficient clearness to be read by the unaided eye at a distance of 6 m. is placed at that distance from the telescope. The instrument is then carefully focused upon it. Then determine how many divisions of this scale as seen by one eye without the telescope exactly cover one of the magnified divisions seen at the same time by the other eye through the telescope. This will give the magnifying power. If we represent the focal lengths of the object glass and eyepiece by F and f respectively, it is shown in geometrical optics that $\frac{F}{f}$ is the magnifying power. Check the results by measuring F and f and calculating the magnifying power.

Form of Record. -- The student is to devise one.

VII. Mapping Spectra and Measuring Wave Length.

81. Problem. — *To map the spectrum of a metal.*

Apparatus. — A **spectrometer** (Fig. 67), and a **Bunsen burner** around the top of which is wrapped a strip of asbestos paper which has been soaked in a strong solution of salt, as sodium nitrate, lithium carbonate, etc., depending on the substance whose spectrum is to be studied. The spectrometer has a prism of flint glass, a telescope, and a tube called the collimator, carrying an adjustable slit at the outer end and a converging lens at the other to render parallel the diverging rays coming from the slit. The supporting table is a circle divided to minutes of arc, and the position of the telescope is read with a vernier.

Directions. — Cover the instrument loosely with a black cloth to cut off all light from the prism except that which passes through the slit. Remove the prism from the stand, turn the telescope till in line with the collimator, and focus the cross threads on the slit. Record the reading of the position of the telescope on the angular scale. Place the prism in position, and also place a common luminous flame in front of the slit. Turn the telescope till a continuous spectrum of the flame is seen; then rotate the prism slowly and notice that the spectrum is displaced. It will be found that the spectrum can be made to move toward the axis of the collimator up to a certain point and then it recedes. Find by trial that position of the prism which places the middle of the spectrum nearest to the collimator axis. This is the position of least deviation for that part of the spectrum, and may be used without serious error as the position of the prism for the several lines to be located.

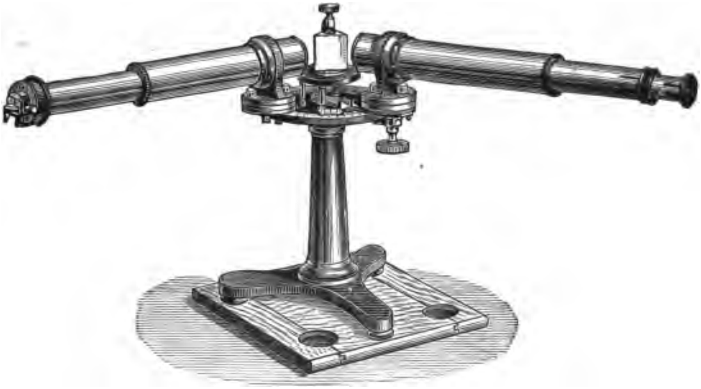

FIG. 67.

Now place the Bunsen burner, with its asbestos tip saturated with sodium nitrate, in front of the slit, focus the telescope with its cross threads exactly on the bright yellow line, which is the spectrum of sodium, and take the reading of the telescope position. The difference of the two readings taken will be the deviation for the sodium spectrum. With instruments of higher power this sodium line appears double, and when the sodium flame is of a very high temperature, as when sodium is placed in the electric arc, several other lines will be found.

By introducing such salts as lithium carbonate, calcium chloride, strontium chloride, barium chloride, etc., into the asbestos cap, the spectrum lines due to lithium, calcium, strontium, etc., can be located.

To construct the map, draw in the notebook a rectangle 10 cm. long and 1 cm. wide to represent the spectrum of the lamp flame. Now find the readings for the limits of the spectrum as seen in the telescope; the difference will be the number of degrees that the 10 cm. represent, from which the value of 1 cm. can be calculated, and also the number of centimeters that correspond to the deviation of each line observed. As each line is located, it should be marked with the chemical symbol of the metal.

Remarks. — Such metals as copper, iron, zinc, etc., can best be vaporized by placing small particles of them in the arc of an arc light, or by using strips of them as the poles of an induction coil. To increase the heating power of the spark, the two poles of the coil should be connected, respectively, to the two surfaces of a Leyden jar. This will make the spark quite short but correspondingly thick. The coil used should give, at least, a spark 5 cm. long.

Form of Record. — Tabulate the telescope readings and the calculated deviation for each line observed.

82. Problem. — *To measure the wave length of sodium light.*

Apparatus. — A **grating** of known constant, three **meter rods**, a **Bunsen burner**, a **piece of sheet tin**, 10 cm. square, in which is a slit 1 mm. wide and 3 cm. long, and a **cylinder of asbestos paper** saturated with sodium nitrate to slip over the mouth of the burner. Suitable gratings of fair quality, ruled on glass, can now be obtained of apparatus dealers. Photographs on glass are equally efficient.

Directions. — Let *MN* (Fig. 68) represent a grating, the dotted openings representing the transparent spaces between the ruled parallel lines; *RS* a screen parallel to *MN* and distant, say, two meters. Let a ribbon of yellow light be incident on the grating. *D* is the central image of this slit on *RS*, *C* and C_1 are the first images of the slit formed by diffraction. Then, by the theory of diffraction, $AC - BC = AE = \lambda$, $AB = d$, the distance between the centers of two contiguous openings, known as the *constant of the grating*. The arc, *BE*, drawn from *C* as a center may be considered as a straight line. Let $AD = a$, and $CD = b$. Then from the similar triangles, *ABE* and *ACD*, we have

$$\frac{AE}{AB} = \frac{CD}{AC}, \text{ or } \frac{\lambda}{d} = \frac{b}{\sqrt{a^2+b^2}}. \quad \text{Hence, } \lambda = \frac{bd}{\sqrt{a^2+b^2}}.$$

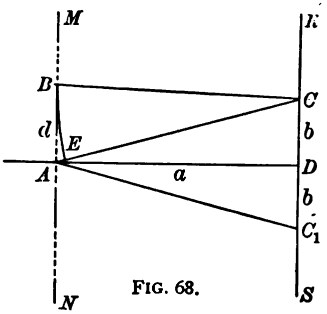

FIG. 68.

To obtain the data for calculating λ, place two meter rods end to end on the table, as *AD*. Over one end, as *A*, support the grating at right angles to *AB*, with its lines vertical to the table. Over the other end, as *D*, support a meter rod at right angles to *AD*, parallel to the table, with its division lines vertical and its middle point exactly at *D*. Fasten to the meter rod the piece of tin, with the slit exactly in line with the 50 cm. mark on the rod. Just back of the slit place the Bunsen burner with its asbestos cap. On looking through the grating at the slit, a number of images of it will be seen, arranging themselves regularly on each side of *D*. Now move white paper sliders along the scale till their edges mark exactly the positions of the images C and C_1. Obtain the distance CC_1; half of this will be the value of b. Substituting the values of a, b, d in the formula for λ, we have the wave length of sodium light.

Remarks. — In order to see the images of the slit, this problem will have to be worked in a semi-darkened room. The work will be facilitated by having two persons engaged in it, one to move the paper sliders while the other is looking through the grating.

Form of Record. — Enter the data as follows : —

	CC_1	b	a	d	λ
	——				
	——				
	——				
Mean	——	——	——	——	——

CHAPTER VII.

HEAT.

83. The Bunsen Burner. — When coal gas is available in the laboratory, the Bunsen burner (Fig. 69) will be found the most convenient and the cheapest source of heat for experimental purposes. It is connected to the gas pipe by means of a rubber tube. To light it, turn on the gas, and as soon as it flows freely from the top of the burner, apply the lighted match. *Do not hold the lighted match over the mouth of the burner while you turn on the gas,* since the gas is likely to take fire down in the tube opposite the lateral openings; that is, the flame "strikes back." The flame then has a reddish orange color on account of the heating of the tube and the imperfect combustion, and the burner will soon be spoiled if the flame is not extinguished. The air supply is usually regulated by a ring near the base. When properly adjusted the flame is nearly colorless. If too much air is admitted for the quantity of gas supplied, the flame is likely to "strike back."

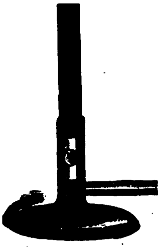
FIG. 69.

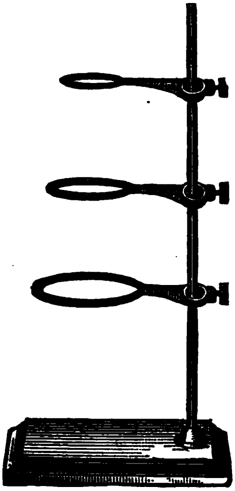
FIG. 70.

84. The iron stand (Fig. 70) consists of an iron rod fixed normally to an iron plate. Clamped to the rod are one or more iron rings for holding vessels. When a glass vessel containing a liquid is to be heated, it is supported by a shallow iron pan, called a sand bath, containing dry sand or a disk of asbestos paper, and is placed on a ring of suitable size clamped at about 5 cm. above the top of the Bunsen burner. *Never place a cold glass vessel on a heated sand bath.*

I. Thermometer.

85. Problem. — *To correct the readings of a thermometer for errors due to the incorrect placing of the freezing point and the boiling point.*

FIG. 71.

Apparatus. — A **mercurial thermometer,** a 4-in. glass **funnel,** a wide-mouthed **bottle,** a liter copper **flask** (Fig. 71), or a liter Florence glass flask, an **iron stand,** a **reading glass,** a wooden **support,** and a supply of clean **ice.**

Directions. — *First.* To test the freezing point, support the funnel in the wide-mouthed bottle. Clamp the thermometer in the arm of the wooden support with its bulb in the funnel. Pack ice shavings around the bulb as far up the stem as the freezing point (Fig. 72). After about ten minutes take the reading of the thermometer, using a reading glass to assist in estimating the tenths of degrees. The correction to be added to the observed freezing point will be found by subtracting the observed value from the true value. For example, if a certain

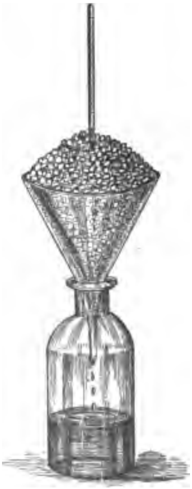

FIG. 72.

Centigrade thermometer reads 0°.8 in ice, then – 0°.8 is the correction to be added.

Second. To test the boiling point, fill the flask about one third full of water, support it on the iron stand, and apply heat. Insert the thermometer within the flask, with its bulb about 2 cm. above the water. If a glass flask is used, support it on a sheet-iron pan containing asbestos paper (Fig. 73) and place within the flask fragments of porous earthenware to prevent "bumping." If the boiling point marked on the stem is some distance above the mouth of the flask, then extend the neck of the flask with a suitable piece of glass tubing or lamp chimney resting upon it. The important point is that the steam must envelop the thermometer stem as far up as the boiling point. After steam has been escaping freely for several minutes, read the thermometer, estimating any fractions of a degree in tenths, and with as little delay as possible read the barometer suspended in the room. The true boiling point on the Centigrade scale will be given accurately to tenths of a degree by the formula $t = 100° - 0°.0375(760 - b)$, in which b is the observed barometric reading expressed in millimeters. The correction to be added to the observed boiling point will be found by subtracting the observed boiling point from the true. For example, let the barometric pressure be 738 mm.; then the true boiling point is 99°.2 C. If the observed boiling point is 98° C., the correction will be 1°.2.

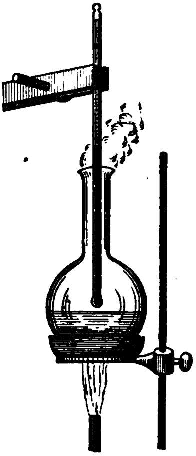

FIG. 73.

Third. To find the correction for any reading, proceed

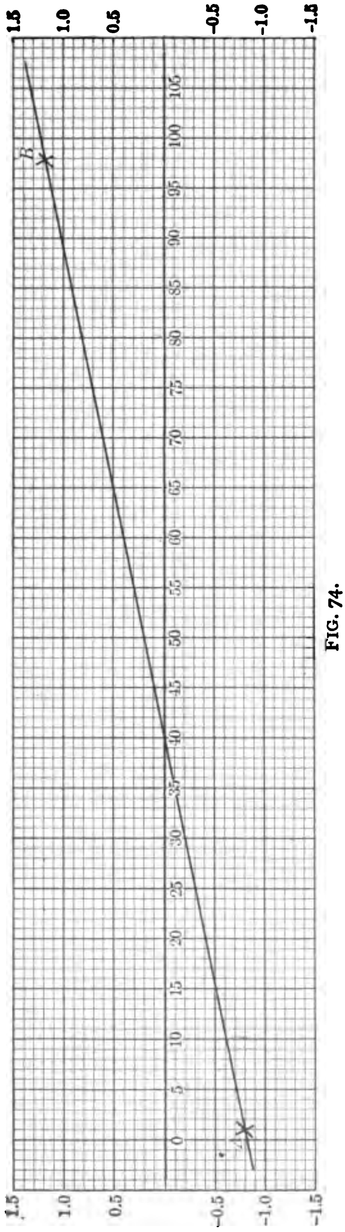

FIG. 74.

as follows: On one of the heavy lines of a sheet of cross-section paper lay off a scale of degrees extending from the freezing point to the boiling point. At a point on this line indicating 0°.8 (using the example above) lay off at right angles to it the distance −0°.8, locating the point, *A* (Fig. 74), and at 98° lay off 1°.2, in the opposite direction, giving the point, *B*. The scale for these measurements should be taken so that a side of a small square represents a tenth of a degree. Join the two points, *A* and *B*, by a straight line. Then the distances of the points of *AB* from the first line are the corrections respectively for the thermometer readings. For example, the correction at 10° is −0°.6, for 40° is 0°, for 70° is 0°.6, etc.

Remarks. — In choosing a dish to hold the broken ice, one with a drainage is necessary, so that the ice water may run off. For this reason the funnel was chosen. A Florence or Wurtz flask may be substituted for the copper vessel. In that case, the smooth glass surface will cause "bumping." This may be largely prevented by covering the bottom of the flask with shot or pieces of the porous cup of a battery. In reading a thermometer, the line of sight must be normal to the scale. The displacement of the zero on heating a thermometer may be seen by returning the thermometer to the ice after taking it out of the steam. It will rarely give the same reading as that given in the first part of the experiment.

Form of Record. — The example used in the directions would be tabulated as follows: —

CENTIGRADE, No. 10.

	FREEZING POINT	BOILING POINT	BAR. PRESS.
True	0°.0	99°.2	738 mm.
Observed . . .	0°.8	98°.0	
Correction . .	– 0°.8	1°.2	

TABLE OF CORRECTIONS.

OBSERVED.	COR.	OBSERVED	COR.	OBSERVED	COR.
0°	– 0°.81	35°	– 0°.10	70°	+ 0°.60
5°	– 0°.71	40°	0°.00	75°	+ 0°.70
10°	– 0°.60	45°	+ 0°.10	80°	+ 0°.82
15°	– 0°.50	50°	+ 0°.20	85°	+ 0°.93
20°	– 0°.40	55°	+ 0°.30	90°	+ 1°.03
25°	– 0°.30	60°	+ 0°.40	95°	+ 1°.13
30°	– 0°.20	65°	+ 0°.51	100°	+ 1°.23

II. Expansion.

86. Problem. — *To find the coefficient of linear expansion of a metallic rod.*

Apparatus. — A **metal rod** about 60 cm. long and 5 mm. diameter, a **thermometer,** a **meter rod,** two square-cornered wooden blocks, and a special device for measuring the expansion (Fig. 75). This special apparatus consists of a wooden frame supporting the rod. The rod is surrounded by a metal tube, closed steam-tight at each end with a thin cork. These corks serve to hold the rod in the axis of the tube. Steam from some suitable generator, as for example, the copper flask of Art. 85, is conveyed by a rubber tube to an arm at one end of the metal tube and escapes through a similar arm at the other end. A rubber tube should be attached to the outlet arm so that the steam and hot water escaping will not come in contact with the wooden frame and expand it. A thermometer is inserted through a stoppered aperture at the middle of the tube. One end of the rod rests firmly against a heavy screw in the end block; this screw is connected by a copper strip to a binding post in the top of this block. The other end of the rod touches the point of a micrometer screw which passes through the heavy wooden block at that end. This screw has a pitch of 1 mm., and the head

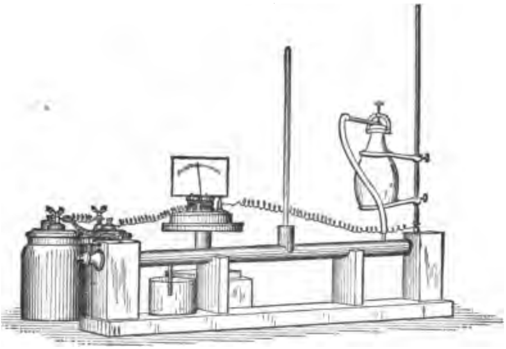

FIG. 75.

is a broad disk, divided on its edge into 100 parts, making the smallest reading 0.01 mm. The head is tangent to a millimeter scale normal to the support. A brass spring presses against this screw and connects it electrically to a binding post in the top of the post. These binding posts are connected electrically through a galvanometer to a battery, the purpose of the galvanometer being to show when the micrometer screw touches the rod.

Directions. — Submerge the rod with its enveloping tube for several minutes in a tank of water maintained at the temperature of the room. The temperature of the rod will now be that of the water. Remove the rod from the tank and place it on the table parallel to the meter rod and between the two square blocks, resting firmly against the meter rod. The difference between the readings of the inner edges of the blocks will give the length of the rod (21). Use a magnifier in making these readings, estimating the fractional part of the millimeter in tenths. Repeat several times, changing the place of the blocks on the meter rod. Now place the rod in the expansion-measuring frame and very slowly turn the micrometer screw, keeping the eye on the galvanometer needle, till the needle deflects. Record the reading of the micrometer head and scale. Turn the screw backward two or three complete revolutions. Now connect the metal jacket to the steam generator, and after steam has been escaping freely from the outlet tube for several minutes, and the thermometer has ceased to rise, turn slowly, as before, the micrometer screw till electrical contact is made, as shown by the deflection of the galvanometer needle. Read the micrometer head and scale and also the thermometer, then turn the screw back, breaking the contact, and discontinue admitting steam.

To obtain the amount of expansion, take the difference between the two micrometer readings. The result will be in millimeters. The expansion divided by the rod's change of temperature will be the expansion for one degree; and this quotient divided by the initial length of the rod, expressed in millimeters, will be the coefficient of expansion.

The experiment should be repeated several times, and the mean taken as the final coefficient. Compare the results with the accepted coefficient (see Table X of the Appendix) and compute the per cent. of error.

Remarks. — See to it that steam does not condense between the ends of the rod and the screw contacts. Keep these dry. Never omit to turn the micrometer screw backward before heating the rod. A failure to do this may bend the rod or spring the frame. Before beginning seriously to solve the problem, practice reading the micrometer in different positions, and also practice turning the screw till electrical contact is secured.

Form of Record. — The following example illustrates the manner of tabulating the data: —

	Length Readings		Length	Initial Temp.	Microm. Reading		Expansion	Final Temp.	Coefficient	Per Cent. of Error
					1st	2d				
			cm.		mm.	mm	mm.			
First trial	10.0	69.19	59.19	18°.3 C.	3.63	4.49	0.86	96°.8 C.	0.00001851	1.02
Second "	15.1	74.20	59.20	18°.1 C.	3.65	4.53	0.88	96°.9 C.	0.00001887	
Third "	20.2	79.38	59.18	18°.2 C.	3.64	4.52	0.88	97°.0 C.	0.00001887	
Fourth "	12.4	70.59	59.19	18°.3 C.	3.66	4.53	0.87	97°.1 C.	0.00001865	
Fifth "	15.0	74.20	59.20	18°.1 C.	3.64	4.52	0.88	96°.9 C.	0.00001887	
Mean . .			59.19						0.00001875	

Alternative Method. — A simple and very efficient apparatus for measuring expansion can be constructed as follows: Procure a tube of metal whose coefficient of expansion is wanted, about 80 cm. long, having a bore of, say, 6 mm. Insert a metal pin 10 cm. from one end of the tube, exactly normal to its surface and projecting 1 cm. on one side and at least 2 cm. on the other. Mount the tube horizontally on a board, supporting its ends by blocks 5 cm. high, with the pin inserted in a hole in the end of one of the blocks to prevent the tube's moving in the direction of its length. At 10 cm. from the other end of the tube a second pin is inserted. This pin does not project through the tube into the supporting block. Firmly clamped to this last supporting block is a micrometer caliper. The height, shape, and position of the block must be such that the square face of the micrometer screw rests against the pin on the top of the tube, the screw being parallel to the tube. Rubber tubes attached to the ends of the metal tube furnish means of passing ice water through it to reduce its temperature, and also of passing steam through it to heat it. The distance between the pins is the length of the tube to be considered in the problem, and may be found with sufficient accuracy by the direct application of the meter rod. The initial temperature is the temperature of the water as it runs from the tube. The final temperature is the temperature of the steam and may be found by calculation from the barometer reading at the time (85). The difference between the two micrometer readings, one taken when the tube was cold and the other when heated, is the expansion. The coefficient will be calculated as already shown. If a caliper is used which indicates contact by the slipping of the milled head, a very accurate measurement of the expansion is possible.

87. Problem. — *To find the coefficient of cubical expansion of a liquid.*

Apparatus. — A **glass tube,** about 15 cm. long and 5 mm. in diameter, closed as square off as possible at one end by fusion, and filled two thirds full of the liquid whose coefficient is sought; a **thermometer**; a **metal tube** 25 cm. long, 3 cm. in diameter, closed at one end with a single perforated stopper fitted with a short glass tube, and at the other by a triple perforated stopper; a **copper flask** with means of heating to supply steam; a **steel scale** reading to fifths of a millimeter, or **vernier caliper.** The apparatus may be mounted as shown in Fig. 76.

Directions. — Measure the length of the glass tube containing the liquid from the inner surface of the closed end to a sharp line marked with a file on the tube near its open end. Close the lower end of the metal jacket by the aid of a short piece of rubber tubing and a pinchcock. By means of a funnel fill the jacket with ice water. After a few minutes draw up the tube till the surface of the liquid is above the cork, and also draw up the thermometer till the top of the mercury is in sight. Record the temperature, and measure with the steel scale or the vernier caliper the distance from the surface of the liquid to the mark on the tube. Return both the tube and the thermometer to their original positions and draw off the water. Now pass steam for several minutes through the jacket till it escapes freely from the lower tube and no

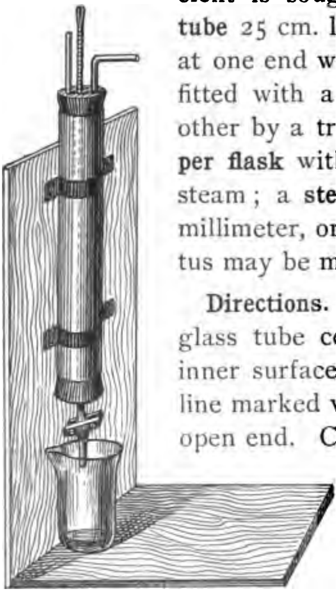
FIG. 76.

change in the reading of the thermometer is observed. Measure and record as before the distance of the surface of the liquid from the mark on the tube and record the temperature. The difference between the last two measurements is the expansion of the liquid cylinder in length, and the difference between the first two measurements is the length of the liquid cylinder at the temperature of the ice water. The expansion divided by the change of temperature will be the mean expansion per degree, and this divided by the initial length of the liquid cylinder will be the apparent coefficient of expansion. Since the volume of the liquid is practically proportional to the length of the cylinder, then this will be the coefficient of cubical expansion. This coefficient is evidently less than the real coefficient on account of the expansion of the glass tube. Hence, to obtain the real coefficient, the apparent coefficient must be increased by that of glass, that is, by 0.0000258. The experiment should be repeated several times, cooling the liquid each time by filling the cylinder with ice water.

Remarks. — For liquids with a boiling point under 100° C., hot water must be used in place of steam, regulating its temperature to suit the liquid. In drawing the tube upward through the cork, give it a twisting motion as you pull. The same thing should be done when moving the thermometer. The measurement of the expansion must be done with the greatest care; a small error will seriously affect the coefficient. A small error in the length of the liquid cylinder is not nearly as serious. The reason for measuring to a line on the tube instead of to the end, is that the end is likely to be uneven, and also a distance between two lines can be more accurately obtained by applying a scale than a measurement from end to end.

Form of Record. — The following example illustrates the manner of tabulating the data: —

Length of Glass Tube	Dist. from Liquid to Line on Tube	Temp. of Water Bath	Dist. from Liquid to Line on Tube	Temp. of Steam Bath	Length of Liquid at Initial Temp.	Exp. of Liquid
cm.	cm.		cm.		cm.	cm.
32.333	15.78	1°.75 C.	15.00	99° C.	16.553	0.78
	15.80	2°.00 C.	15.03	99° C.	16.533	0.77
	15.81	2°.00 C.	15.04	99° C.	16.523	0.77

Change of Temp.	Apparent Coefficient of Expansion	Coeff. cor'tn for Expansion of Glass Tube	Accepted Coefficient	Per Cent. of Error
97°.25 C.	0.0004845			
97°.00 C.	0.0004801			
97°.00 C.	0.0004804			
	0.0004817	0.000507	0.000526	3.6

An Alternative Method: Apparatus. — A **density bottle** (58), unadjusted, of about 50 cm.^{3} capacity, a **balance,** and a **set of weights.**

Directions. — Clean the bottle thoroughly, dry, and find its mass (31). Fill the bottle with the liquid whose coefficient of expansion is sought, the temperature of the liquid being that of the room, weigh, and calculate the mass of the liquid. Now place the bottle with the contents in a water bath and gradually raise the temperature to a point several degrees less than the boiling point of the liquid. Note the temperature carefully, remove the bottle from the bath, carefully dry the outside, weigh, and then find the mass. This mass will be less than the first, because of the overflow through the perforated stopper as the liquid expanded. This last mass may be considered as the measure of a quantity of the liquid, which, when heated through a range of temperature indicated by the difference of the two thermometer readings, will expand by an amount equal to the difference of the two masses. Hence, the mean apparent coefficient of expansion is equal to the difference of the two masses of the liquid divided by the product of the mass of the liquid at the higher temperature by the change of temperature. To obtain the true coefficient, the apparent coefficient must be increased by that of the glass, that is, by 0.0000258.

Form of Record. — Article 31 will suggest the proper manner of recording the data.

88. Problem. — *To measure the coefficient of expansion of air under a constant pressure.*

Apparatus. — A **thermometer**, a large **glass jar**, and a **J-shaped glass tube** of about 5 mm. bore, the short arm, *A* (Fig. 77), closed square off by fusing a short piece of glass rod in the end. The short arm should be about 40 cm. long, and the long arm 60 cm. A glass tube, *F*, is provided to slide freely in the open arm, *B*. A broad metal band, *D*, encircles both arms near the end of the closed branch. Through loops on this band a wire, *C*, slides with sufficient friction to stay wherever placed. This wire terminates in a metal index, *I*, at right angles to the wire. A fine scale will be necessary for measuring distances on the closed arm. This may be either a steel scale or the mirror scale of a Jolly balance[1] (32).

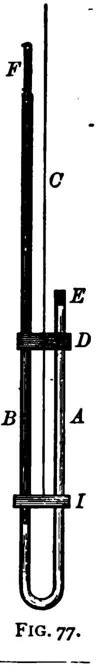

FIG. 77.

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[1] This method is due to Mr. N. H. Williams of the University of Rochester.

Directions. — Pour into the J-tube sufficient sulphuric acid to fill both arms to a level of about 15 cm. when the glass plunger is inserted and extends nearly to the curve. Place the apparatus in a jar of ice water, and after several minutes adjust the glass plunger till the acid is at the same level in each branch, mark the level of the acid with the sliding index, and note the temperature of the water bath. Remove the apparatus from the bath, and with the scale measure very carefully the length of the air column.

Repeat the experiment with a water bath of about 10° C., then 20° C., and so on till as hot water as can be safely poured into the jar is used.

Use the two extreme temperatures and compute the mean expansion per degree. This divided by the volume or length of the air column at the first temperature will be the apparent coefficient of expansion. To correct for the expansion of the glass add 0.0000258.

Plot a curve, using the lengths of the air columns as abscissæ and the corresponding temperatures as ordinates. The points should all be situated on or about a straight line. Produce this line till it cuts the axis of Y. This point of intersection will be about 273° below the origin.

Remarks. — The sulphuric acid should be poured into the tube some time previous to performing the experiment, so that the moisture of the air in the closed arm may be absorbed. Capillarity and parallax are fruitful sources of error in taking the readings, and any lack of uniformity of cross-sectional area of the tube would vitiate the results. It is not necessary that the lowest temperature be 0° in order to obtain the coefficient, as the following will show: Let $v_1 =$ the volume at t_1, $v_2 =$ the volume at t_2, and $v_0 =$ the volume at 0°. Then $v_1 = v_0(1 + at_1)$, where a is the coefficient of expansion, and $v_2 = v_0(1 + at_2)$. Eliminating v_0, we have $\frac{v_1}{v_2} = \frac{1 + at_1}{1 + at_2}$, from which $a = \frac{v_2 - v_1}{v_1 t_2 - v_2 t_1}$.

Form of Record. — The results may be tabulated as follows: —

VOLUME OF AIR READINGS.			CORRES. TEMP.	TOT. CHANGE OF VOL.	TOT. CHANGE OF TEMP.
Top.	Bottom.	Volume.			
——	——	——	——		
——	——	——	——		
etc.	etc.	etc.	etc.	——	——

Apparent Mean Coef. of Exp. ——. Accepted Coef. ——.
Corrected for Exp. of Glass ——. Per Cent. of Error ——.

III. Melting and Boiling Points.

89. Problem. — *To determine the melting point of such substances as paraffin, beeswax, spermaceti,* etc.

Apparatus. — A **thermometer**, glass **beaker** of a liter capacity, **iron stand, Bunsen burner,** a **test tube** about 25 mm. in diameter, some capillary tubing about 1 mm. bore. Suitable capillary tubes may be prepared by softening a piece of German soft glass tubing in a Bunsen flame and then drawing it out.

Directions. — Melt some of the substance in a suitable vessel and partly fill the capillary tube by suction. Close the end with a plug of plaster of Paris. There should be a fine opaque thread of the substance in the tube. Tie the tube to the thermometer with that part containing the substance resting against the bulb. Insert the thermometer in the test tube, so as not to touch its walls, close the mouth loosely with cotton wool to prevent air currents, and hold the whole in a beaker of hot water. Closely watch the filament in the capillary tube, and on its becoming transparent record the reading of the thermometer.

Lift the test tube out of the bath and record the temperature at which the filament becomes opaque. Repeat several times, and take the mean of all the results.

Remarks. — The following example illustrates the manner of tabulating the results: —

	SUBSTANCE	MELTING POINT	CONGEALING POINT	PROBABLE MELTING POINT
First trial	Spermaceti	46°.0 C.	45°.5 C.	44°.4 C.
Second "	"	43°.5 C.	42°.2 C.	
Third "	"	45°.0 C.	44°.5 C.	
	Mean . .	44°.8 C.	44°.1 C.	

90. Problem. — *To find the boiling point of a liquid.*

Apparatus. — A **test tube** about 25 mm. in diameter, a **double-perforated cork,** a **thermometer, beaker, iron stand, Bunsen burner,** and a **barometer.**

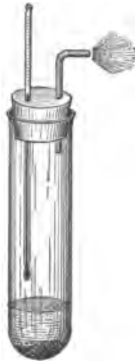

FIG. 78.

Directions. — Insert the thermometer in one of the holes of the cork, and a glass tube bent at right angles in the other to serve as an escape for the vapor (Fig. 78). Pour into the test tube about 15 cm.^{3} of the liquid to be tested, and adjust the thermometer so that its bulb is at least 2 cm. above the liquid. Hold the tube in a beaker of hot water. The temperature of the heating bath should not be many degrees above the boiling point of the liquid. The test tube should be submerged in the bath as far as the liquid reaches up the test tube. When the vapor escapes freely from the bent tube, note the reading of the thermometer and also that of the barometer. Since the boiling point of a liquid increases 0°.0375 C. for an increase of 1 mm. in the atmospheric pressure, hence, to obtain the boiling point at a pressure of 76 cm., the observed boiling point must be increased by $0^\circ.0375(760 - b)$, in which b is the barometric pressure expressed in millimeters at the time that the experiment was performed.

Remarks. — Liquids whose boiling points are above 100° C. will have to be heated in a bath of linseed oil. Very combustible liquids, like ether, must be boiled at a considerable distance from lighted lamps. If pieces of crumpled tin foil or of porous, unglazed earthenware be placed in the test tube, the boiling will be more regular.

Form of Record. — The following example illustrates the manner of tabulating the results: —

	LIQUID	OBSERVED BOILING PT.	BAROMETER PRESSURE	BOILING PT. AT 76 CM.	ERROR	PER CENT. OF ERROR
1st trial	Benzol	80°.0 C.	741.93 mm.			
2d "	"	79°.9 C.				
3d "	"	80°.0 C.				
	Mean .	79°.97 C.		80°.65	0°.25	0.31

91. Problem. — *To determine the dew-point.*

Apparatus. — A **nickel-plated beaker** of one-fourth liter capacity (Fig. 79), a **thermometer**, and **ice**.

Directions. — Polish the outside of the beaker till it has a mirror surface. Fill the beaker half full of water, at the temperature of the room. Add small pieces of ice, stirring thoroughly with the thermometer. Note the temperature of the water on the first appearance of mist on the mirror surface of the beaker. Then quickly remove any unmelted ice and record the temperature on the disappearance of the mist. The mean of these results may be considered as the dew-point. Repeat the experiment several times.

Remarks. — In conducting the experiment be careful not to breathe upon the beaker.

Form of Record. — The results may be tabulated as follows: —

	TEM. ON APPEARANCE OF MIST	TEM. ON DISAPPEARANCE OF MIST	DEW-POINT
First trial . .	——	——	——
Second " . .	——	——	——
Third " . .	——	——	——
	etc.	etc.	etc.
		Mean	——

IV. Calorimetry.

92. A **calorie** is the quantity of heat necessary to raise the temperature of a gram of water 1° C.

93. The **thermal capacity** of a body is the number of calories of heat necessary to raise its temperature 1° C.

94. The **specific heat** of a substance is the number of calories of heat necessary to raise the temperature of one gram of the substance 1° C. Hence, the mass of a body in grams multiplied by the specific heat, and that by the change of temperature, will equal the heat gained or lost by the body, as the case may be.

95. A **calorimeter** is an apparatus used for measuring the quantity of heat. For the purposes of this book, a small metallic beaker (Fig. 79) is all that is necessary. Preferably it should be nickel-plated, and when necessary to cut down radiation it may be wrapped with asbestos paper.

FIG. 79.

96. The **water equivalent** of a vessel is the number of grams of water that has the same thermal capacity as the vessel. It is numerically equal to the thermal capacity of the vessel, and hence may be found by multiplying the mass of the vessel in grams by the specific heat of the substance of which the vessel is made (Table XIII of Appendix).

97. Problem. — *To determine the specific heat of a solid.*

Apparatus. — A **beaker** of $\frac{1}{4}$ l. capacity to serve as a calorimeter, a **thermometer**, an **iron stand,** a **Bunsen burner,** a **test tube** about 25 mm. in diameter, and a second beaker in which to heat water.[1]

Directions. — Cut enough of the solid into small pieces to fill the test tube about half full. Find the weight of these pieces. Insert a thermometer in the test tube and close the mouth loosely with a plug of cotton wool to prevent air currents. Suspend the test tube for several minutes by a wire handle in a beaker of boiling water. Weigh the calorimeter and compute its water equivalent (96). Observe the temperature of the room at the place of work. Fill the calorimeter half full of water at this temperature and find its weight. When the thermometer in the test tube ceases to rise, note its reading, and as quickly as possible pour the contents of the tube into the calorimeter. Stir the water thoroughly, watching the reading of the thermometer, and note the temperature when at its highest point. The water in the calorimeter increased by the water equivalent of the calorimeter, multiplied by the increase of temperature, will be the heat gained. This must equal that lost by the given body, that is, the mass of the body multiplied by its specific heat and that by its change of temperature. Hence, the specific heat will equal the heat gained divided by the product of the mass of the body by the change of temperature. There is an error in the work due to radiation, and in repeating the experiment, that error may be reduced, if not wholly removed, by giving the water in the calorimeter a temperature below that of the room by an amount equal to half of the increase in the trial just made. Then the water will at first be heated by the room, and during the last part of the change the water will radiate heat to the room, the gain in the first half being nearly offset by the loss in the last half.

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[1] It will save time to have in the room two large vessels of water, one at the temperature of the room and the other at or near the boiling point.

Remarks. — In repeating the experiment, the pieces of the solid must be carefully dried before reheating. The calorimeter should stand on a mat of asbestos paper, should be wrapped with two or three layers of such paper, should be at some distance from the Bunsen flame, and should not be touched by the hands after the temperature observations begin.

Form of Record. — Enter results as follows: —

Temperature of room	——	——	——
Mass of calorimeter	——	——	——
Specific heat of calorimeter	——	——	——
Water equivalent of calorimeter . . .	——	——	——
Mass of solid used	——	——	——
Temperature of solid when heated . .	——	——	——
Mass of calorimeter and water . . .	——	——	——
Mass of water	——	——	——
Initial temperature of water	——	——	——
Final temperature of water	——	——	——
Specific heat of substance	——	——	——
Mean			——
Accepted value			——
Per cent. of error			——

98. Problem. — *To determine the specific heat of a liquid.*

Apparatus. — Same as in Art. 97.

Directions. — Proceed as in Art. 97, with the exception that a known mass of the liquid is substituted for the water. The specific heat of the metal may be found in Table XIII of the Appendix. If the mass of the metal is m, the specific heat s, its change of temperature t, then the heat lost by the metal will be mst. If the mass of liquid be m', the specific heat s', its change of temperature t', then the heat gained by the liquid will be $m's't'$. If the water equivalent of the beaker is m'', then the heat gained by the beaker is $m''t'$. Therefore, $mst = m's't' + m''t'$, from which

$$s' = \frac{mst - m''t'}{m't'}.$$

Remarks. — The metal must not be heated to the boiling point of the liquid.

Form of Record. — Enter results as follows: —

Temperature of room	—	—	—
Mass of calorimeter	—	—	—
Specific heat of calorimeter	—	—	—
Water equivalent of calorimeter . . .	—	—	—
Mass of metal	—	—	—
Specific heat of metal	—	—	—
Mass of liquid and calorimeter . . .	—	—	—
Temp. of metal when heated	—	—	—
Initial temp. of liquid	—	—	—
Final temp. of liquid	—	—	—
Specific heat of liquid	—	—	—
Mean			—
Accepted specific heat			—
Per cent. of error			—

99. The **heat of fusion** of a solid is the number of calories of heat required to melt one gram of that substance without raising its temperature.

100. The **heat of vaporization** of a substance is the number of calories of heat required to change one gram of that substance at its boiling point into vapor at the same temperature.

101. Problem. — *To determine the heat of fusion of water.*

Apparatus. — A **calorimeter**, a **thermometer**, clean **ice**, and **hot water.**

Directions. — Weigh the calorimeter and compute its water equivalent (96). Break the ice into pieces of the size of a small walnut. Fill the calorimeter about half full of water, whose temperature is about 40° C. and find its mass. Determine its temperature exactly, then wipe the pieces of ice with a cloth and introduce them into the calorimeter as quickly as possible. The amount added should be such that when melted the temperature of the water will be about as much below that of the room as it was above before adding the ice. A trial experiment may be necessary to determine the amount. A degree or two either way will matter but little, the point being to correct approximately for the radiation by balancing the loss due to radiation when the water was warmer than the room against the heating of the water by the room during that period when the temperature of the room was higher than the water. When the ice is all melted stir very thoroughly and then note the temperature. Now find the mass of the calorimeter and contents. If m be the mass of water, m' the water equivalent of the calorimeter, and t the water's change of temperature, then $(m+m')t$ will be the heat lost by the water. If m'' is the mass of ice, L its heat of fusion, and t'' the temperature of the water when the ice was melted, then $m''L+m''t''$ will be the heat taken up by the ice in melting and in heating the resulting water to the temperature observed at the conclusion of the experiment. Then, as the heat lost by the water equals the heat taken up by the ice, we have $(m+m')t=m''L+m''t''$, from which $L=\frac{(m+m')t-m''t''}{m''}$.

Remarks. — The ice is wiped to reduce the error caused by introducing water. The water must be thoroughly stirred before taking its temperature on account of its poor thermal conductivity. If the calorimeter be wrapped with asbestos paper, the heat radiation will be reduced. For fear of breaking the thermometer, a glass rod may be used as a stirrer. The correction for the heat imparted by the rod to the ice may be computed by weighing the rod and then estimating the amount that was in the water; this mass multiplied by the specific heat of the glass and this by the change of temperature will be the heat that comes from the rod, and may be allowed for in the calculation.

Form of Record. — Enter results as follows :—

Mass of calorimeter	—	—	—
Specific heat of calorimeter	—	—	—
Mass of water and calorimeter at beginning .	—	—	—
Temperature of water at beginning	—	—	—
Temperature of water at end	—	—	—
Mass of water and calorimeter at end . . .	—	—	—
Mass of water in calorimeter	—	—	—
Mass of ice melted	—	—	—
Heat of fusion	—	—	—
Mean			—
Accepted value			—
Per cent. of error			—

102. Problem. — *To determine the heat of vaporization of water.*

Apparatus. — A **copper** or **glass flask** of 1 l. capacity, fitted with a delivery tube (Fig. 80) to convey steam from the flask to a beaker of $\frac{1}{4}$ l. capacity serving as a calorimeter. In the delivery tube is a water trap to retain the water of condensation, so that steam alone reaches the calorimeter. The trap is provided with a tube, not shown in the figure, for emptying it without removing the stopper. This tube is closed by means of a short piece of rubber tubing and a pinchcock. A wooden

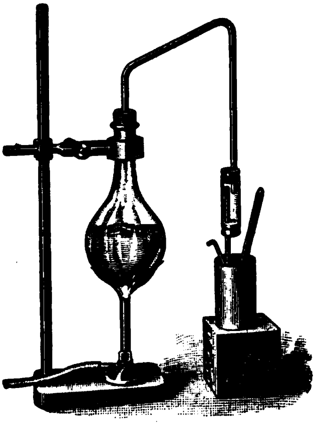

FIG. 80.

screen should be placed between the iron stand and the calorimeter to cut off radiation from that source.

Directions. — Weigh the calorimeter and compute its water equivalent (96). Fill the calorimeter about half full of water at a temperature of about 10° C. below that of the room, and find the mass. Now introduce a strong jet of steam from the delivery tube of the flask. Let the delivery tube reach nearly to the bottom of the water. Stir the water constantly and thoroughly with the thermometer, and when the temperature has risen as many degrees above that of the room as it was below at the beginning of the introduction of steam, remove the steam delivery tube as quickly as possible. Stir the water thoroughly for two or three seconds, then read the thermometer and record. Weigh the calorimeter and its contents to obtain the quantity of steam condensed. Suspend the thermometer within the flask to obtain the temperature of the steam. The computation of the heat of vaporization is similar to that of Art. 101.

Remarks. — By raising the temperature of the water from a point below that of the room to one an equal number of degrees above that of the room, the error due to radiation is largely eliminated. The temperature of the room is found by suspending a thermometer above the table. The most serious error entering into the data will be found in getting the correct temperature of the heated water, on account of the difficulty of thoroughly mixing the steam water with the water placed in the calorimeter. The absorption of heat by the thermometer itself is a source of error, and might be corrected by finding its thermal capacity either experimentally or by computation from its mass and specific heat.

Form of Record. — Enter results as follows: —

Mass of calorimeter	——	——	——
Specific heat of calorimeter	——	——	——
Mass of water and calorimeter at beginning .	——	——	——
Initial temperature of water	——	——	——
Final temperature of water	——	——	——
Mass of calorimeter and contents	——	——	——
Water equivalent of calorimeter	——	——	——
Mass of water in calorimeter	——	——	——
Mass of steam introduced	——	——	——
Heat of vaporization	——	——	——
Mean			——
Accepted value			——
Per cent. of error			——

CHAPTER VIII.

MAGNETISM AND ELECTRICITY.

I. Magnetism.

103. Problem. — *To find the position of the poles of a bar magnet.*

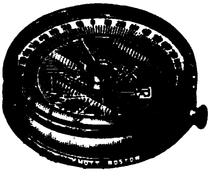

FIG. 81.

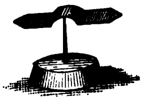

FIG. 82.

Apparatus. — A **bar magnet**, a **sheet of paper** fastened with brass tacks to a piece of board, a **pocket compass** (Fig. 81), and several brass pins. In place of a compass a piece of watch spring about 3 cm. long, bent, magnetized, and mounted on a needle as shown in Fig. 82, may be used.

Directions. — Ascertain by means of the compass which is the north pole of the magnet and mark it. Place the magnet at the center of the sheet of paper and trace its outline. Mark that end of the outline which is to represent the north pole. Place the compass off one corner of the north pole of the magnet (Fig. 83) and rotate the board carrying the apparatus till the compass points north. The north pole of the magnet is now in the magnetic meridian. Stick two pins vertically in the paper, one on each side of the compass, the line of the two being exactly in that of the needle. Now move the compass to the adjoining corner and proceed as before. Remove the magnet and draw straight lines exactly through each pair of points marked by the pins. The intersection of these lines will locate the pole of the magnet. In like manner locate the south pole. Either insert this sheet of paper in the notebook or transfer the figure by placing the paper over the notebook page and pricking with a needle the corners of the magnet and the points where the pins were situated. Locate the poles of a thick bar magnet and also of a long slender one. What difference do you find in the position of poles relative to the ends of the magnet?

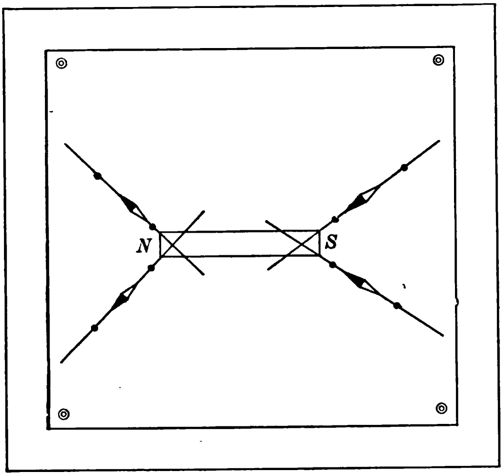

FIG. 83.

Remarks. — The compass does not point exactly toward the pole on account of the action of the other pole of the magnet. The effect of this pole will be reduced by placing the compass more toward the end than toward the side of the magnet. Try more than two positions of the compass for each pole and see if all the lines pass through a common point. What effect would it have not to rotate the board till the compass points north?

Form of Record. — The record will consist of the diagrams and such measurements as may be necessary to bring out the differences in the position of the poles in the different cases tried.

104. Problem. — *To measure the relative magnetic transparency of thin sheets of various substances.*

Apparatus. — A **magnetoscope** (Fig. 84), **thin sheets** of wood, zinc, brass, and iron, each about 10 cm. square and all of equal thickness, a **watch** provided with a second hand, a **bar magnet**, and a suitable support for the magnet and the thin sheets. The support may be made out of an empty chalk box, the magnet being supported on the top of the box and the thin sheets in clips on the end (Fig. 85). In Art. 136 there is given a description of a simple magnetoscope.

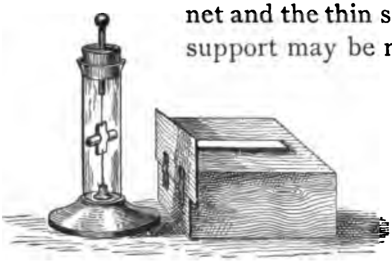

FIG. 85.

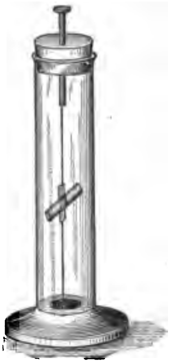

FIG. 84.

Directions. — Adjust the magnetoscope so that it points north and south. Set its magnet vibrating through a small arc by bringing a bar magnet momentarily near it. Count the number of vibrations it makes in 30 sec. During the counting of these vibrations all magnets must be kept at a considerable distance from the magnetoscope. The magnet vibrates under the action of the earth's magnetism, and the number of these vibrations will serve as a measure of the intensity of the earth's field at that place. Now support the bar magnet with its axis in line with that of the magnetoscope needle, opposite poles being placed adjacent and separated by 3 or 4 cm. This distance must remain constant during the work. Again set the magnetoscope needle vibrating and determine the number of single vibrations made in 30 sec. If the needle vibrates too fast for accurate counting, move the bar magnet farther away. Now place one of the thin sheets between the magnet and the magnetoscope and determine the number of vibrations. In this way test the various sheets, and in the case of the iron, also test one of considerably greater thickness. In determining the number of vibrations of the magnetoscope, at least three trials of each case should be made and the mean obtained.

The vibrating needle is a pendulum, and hence the force which actuates it is proportional to the square of the number of vibrations. In the first case, this force was the magnetic force of the earth, or H; then $H \propto n^2$, n being the number of single vibrations made in 30 sec. under the influence of this force. In the second case, the force is that of the magnet and earth combined, or $F + H$; then $(F + H) \propto N^2$, N being the number of single vibrations made in 30 sec. under the influence of the two forces. Hence, $F \propto (N^2 - n^2)$, that is, to obtain the measure of the force of the magnet through the various sheets, subtract the square of the number of vibrations due to the earth alone from the square of those due to the magnet and earth combined. The relative magnetic transparency will be found by dividing through by the value obtained for $N^2 - n^2$ when no substance intervenes.

Remarks. — No change, either of position or adjustment, should be made in the magnetoscope or the magnet after the observations begin, since either will change the rate of vibration. The solution of the problem assumes that glass offers no obstruction to the magnetic force. This could be proved by using a mounted magnetic needle as a magnetoscope, and placing a plate of glass between the needle and the bar magnet. The discussion should make a close comparison of the results, pointing out any differences that exist and what they signify.

Form of Record. — The following example illustrates the manner of tabulating the results: —

EARTH	WITHOUT SUBSTANCE	ZINC	WOOD	IRON	IRON (thicker)
22	68	69	68	50	47
24	70	68	70	48	46
23	68	70	69	50	46
Mean 23	68.7	69	69	49.3	46.3
529	4719.7	4761	4761	2430.5	2143.7
F or $N^2 - n^2$	4190.7	4232	4232	1901.5	1614.7
Rel. trans.	1.00	1.01	1.01	0.45	0.39

105. Problem. — *To compare the strength of the poles of a bar magnet.*

Apparatus. — A **bar magnet**, a **magnetoscope**, and a **watch** with a second hand.

Directions. — Adjust the magnetoscope so that the needle points north and south, and then ascertain the number of single vibrations made by the needle in 30 sec., that is, find the strength of the earth's field at that place. To start the needle oscillating, bring a bar magnet near for a moment, but while counting, all magnets must be removed to a distance of at least a meter. Support the bar magnet vertically, with the north pole in the magnetic axis of the needle, opposite its south pole, and from 3 to 5 cm. distant, and determine the number of vibrations as before. In like manner, test the south pole. At what distance should the south pole be from the magnetoscope? Why? Make at least three counts in each case and find the mean.

As shown in Art. 104, the force of the magnet is obtained by subtracting the square of the number of vibrations due to the earth from the square of the number due to the magnet and earth combined. The relative strength of the two poles will be the ratio of the north pole to the south.

Remarks. — The magnetoscope needle should be started vibrating each time through the same sized arc. The position of the poles should be found by Art. 103. By supporting the magnet in a vertical position, then those parts of the magnet not in the horizontal plane of the needle exert no force on the needle.

Form of Record. — The following example illustrates the manner of tabulating the results:—

	EARTH	NORTH POLE	SOUTH POLE
	23	65	60
	22	64	59
	24	66	61
Mean	23	65	60
	529	4225	3600
	$N^2 - n^2$	3696	3071

North Pole : South Pole = 3696 ÷ 3071 = 1.203.

106. Problem. — *To determine the distribution of magnetism along a bar magnet.*

Apparatus. — A slender **bar magnet**, a **magnetoscope**, a **watch**, and a **clamp support**. A suitable slender magnet for the experiment may be made from a piece of drill steel about 4 mm. in diameter and 16 cm. long. To magnetize it, wind on it, evenly and smoothly, from end to end, a layer of insulated copper wire, about No. 20, and connect to a battery of two or three cells. The magnet should be marked off into equal parts of 2 cm. each by fine lines of white paint, and the two parts at each end should be divided each into centimeter spaces. There will then be thirteen points on the magnet, including the ends.

Directions. — Find the measure of the earth's field as in Art. 104. Now ascertain which is the north pole of the slender magnet; then support the magnet vertically with the north pole on the south side of the magnetoscope and the end of the magnet exactly in the axis of the magnetoscope needle. The magnet should be about 4 cm. distant from the magnetoscope needle, and *throughout the entire experiment this distance must be constant.* Determine the number of vibrations in 30 sec., making at least three counts. Bring in succession each of the thirteen points into the axis of the magnetoscope and determine the number of vibrations. When, however, the number becomes less than that due to the earth alone, the magnet must be transferred to the other side of the magnetoscope, so that attraction, and not repulsion, may actuate the instrument. The point at which the change was made must be indicated in the record.

The measure of the strength of magnetism at each of the points is found as in Art. 104. The relative strength is best shown graphically. On a page of cross-section paper draw a heavy straight line 16 cm. long, lengthwise of the page, and on its middle line, to represent the magnet. Mark one end of it *N* and the other *S*. Also mark points on the line to represent the points that were tested. The magnet should be so located on the page that these points fall on the heavy ruled lines of the cross-section paper. Designate as *O*, the line on which the magnet is drawn, and lay off each way from *O* a scale, making each centimeter represent 100 or 200, according to the size of the numbers to be plotted. Now locate points on the page whose distances from the magnet, as measured on the scale chosen, represent the magnetic strength of these points, those points belonging to the north end of the magnet being located to the right of the magnet line, and those belonging to the south end to the left. Sketch a smooth curve through these points, and we have a graphic representation of the change in external magnetism exhibited by the magnet as one passes from one pole to the other. The magnetic poles will be shown by the points on the curve most remote from the magnet, and the magnetic equator by the point at which the curve crosses the magnet.

Remarks. — An important source of error is found in not maintaining the magnet at a constant distance from the magnetoscope. Should the magnetoscope vibrate too rapidly for accurate counting, place the magnet farther away. In choosing the scale for the curve, it is advisable to plan it so that the curve shall not be more than 5 cm. removed from the line representing the magnet at the point representing greatest strength.

Form of Record. — The following example illustrates the manner of tabulating the results : —

	EARTH	NORTH END						SOUTH END						
		1	2	3	4	5	6	7	8	9	10	11	12	13
First count .	16.0	30.0	32.5	30.5	26.0	21.5	16.5	17.5	19.5	22.5	26.0	30.5	32.5	31.0
Second " .	16.5	29.5	32.0	29.5	25.5	21.0	17.0	17.0	20.0	22.0	25.5	31.0	33.0	30.5
Third " .	17.0	31.0	32.5	30.0	25.0	21.0	17.0	17.5	19.5	23.0	26.5	30.5	32.5	31.5
Mean . . .	16.5	30.2	32.3	30.0	25.5	21.2	16.8	17.3	19.7	22.5	26.2	30.7	32.7	31.0
	272	912	1043	900	650	449	282	299	388	506	686	942	1069	961
Relative strength		640	771	628	378	177	10	27	116	234	414	670	767	689

107. Problem. — *To map the magnetic field of a magnet.*

Apparatus. — Two **bar magnets**, each about 8 cm. long, a **horseshoe magnet** with parallel branches, a **soft iron ring** about 2.5 cm. in diameter, **iron filings**, and a mounted **magnetic needle** about 1.5 cm. long, the support of the needle being 1 cm. high. The iron filings should be kept in a two-ounce wide-mouthed bottle, the cork of which has a hole through it at least 2 cm. in diameter, this hole having fastened over it a disk of brass wire cloth, having about 12 meshes to the centimeter. The magnetic needle may be made of a piece of watch spring about 2 cm. long, bent as shown in Fig. 82, and mounted on a piece of a sewing needle driven into a small lead disk for a base.

Directions. — *First Method.* Fasten with brass tacks a sheet of paper to a small square board. Place the magnet on the center of the sheet and trace its outline with a pencil. Stand the small compass needle near the north pole of the magnet, turn the board till the needle points north and then as soon as the needle comes to rest mark with a pencil a point exactly under each of its poles. Move the needle farther away from the magnet till the south pole coincides with the previously marked north pole after the board is again turned so that the needle points north. Again mark the position of the pole. Proceed in this way till the south pole of the magnet is reached. A smooth curve traced through these points will represent a line of force. In this way trace several lines branching out from either pole.

Map the magnetic field in each of the following cases: (*a.*) *Single bar magnet.* (*b.*) *Two bar magnets placed parallel to each other with like poles adjacent.* (*c.*) *Two bar magnets placed parallel to each other with unlike poles adjacent.* (*d.*) *A bar magnet placed in the magnetic meridian, with north pole to the north, and not to be moved while mapping the field, so that the combined fields of the earth and the magnet may be obtained.* (*e.*) *A horseshoe magnet.*

Second Method. Cut a mortise in a board, so that the magnet placed in it will lie with its top surface flush with the face of the board. Pin a sheet of blue print paper over the magnet, and sift iron filings evenly on it, tapping the board to facilitate the moving of the filings into the regular curves which mark the lines of force of the field. This should be done at a part of the room where the light is quite subdued. Now move the board, without jarring, into a strong light, preferably direct sunlight, for a few minutes. When the printing is completed, brush off the filings with a brush or clean cloth and develop the print in the usual way.

Map the same cases as in the first method except that for Case (*d*) substitute a bar magnet with a soft iron ring placed about 1 cm. distant from one pole.

Remarks. — The first method of tracing the lines of force is somewhat lengthy, but the results are more accurate than the method with filings, since the friction of the needle on its point is much less than that of the filings on the paper. Then, too, the first method eliminates the effect of the earth's field, and the small needle disturbs the field less by its permeability than the considerable quantity of iron filings distributed over the paper does. It might be well to study some of the cases by the first method and others by the second method. All the diagrams should be neatly fastened in the notebook, properly marked, and descriptions appended. In discussing these fields, special attention should be given to bringing out the properties of these lines as developed by the experiment. 1. Are the lines straight or curved? Why? 2. Do they cross one another, or merge into one another? 3. Are they open or closed curves? 4. Are they uniformly distributed or are there crowded places? 5. Do the fields suggest any explanation of magnetic attraction, magnetic repulsion, and magnetic induction?

II. Electricity.

108. Problem. — *To determine the kind of electrification of a charged body.*

Apparatus. — An **electroscope** (Fig. 86), a **rod of flint glass** about 12 mm. diameter and 30 cm. long, a **silk cloth**, a **stick of sealing wax**, a piece of **flannel**, and several substances to be tested.

Directions. — Charge the electroscope by induction from the body of unknown electrification. This is done by holding the finger of one hand in contact with the ball of the electro-

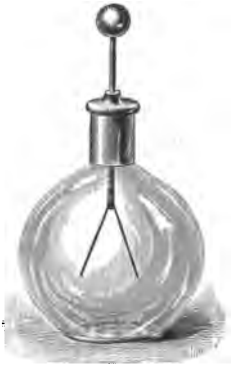

FIG. 86.

scope, and then bringing the electrified body near the ball but not touching it. While the electrified body is near the ball remove the finger from it, and afterwards remove the electrified body; the leaves of the electroscope will now diverge, being charged oppositely to this body. Explain. The intensity of the induced charge should be such as to separate the leaves about 1 cm. This is regulated by the distance that the charged body is held from the ball; the nearer the greater the induced charge, and conversely. Now bring the electrified glass rod ($+E$) gradually near the electroscope ball, and observe whether the divergence of the leaves is increased or diminished. Repeat the test using the electrified sealing wax ($-E$). The charge of the electroscope is of the same sign as that which increases the divergence, and hence that of the body under examination is of the opposite sign.

Remarks. — The first movement of the leaves as the electricity of known sign is brought near the electroscope must be recorded. If the glass of the electroscope becomes electrified, the indications of the instrument will be unreliable. Substances suitable for this experiment are smooth glass, paper, fur, flannel, silk, wood, metal rod with glass or rubber handle, hard rubber, rosin, rough glass, cotton, paraffin, sealing wax, wooden rod heavily coated with shellac varnish, wooden rod coated with sulphur.

Form of Record. — Results may be tabulated as follows: —

NAME OF SUBSTANCE	WITH WHAT RUBBED	BEHAVIOR OF LEAVES UNDER $+E$	BEHAVIOR UNDER $-E$	KIND OF ELECTRIFICATION
Vulcanite etc.	Fur etc.	Diverg. increased etc.	Diverg. lessened etc.	Negative etc.

109. Problem. — *To ascertain the effect of an electric current on a mounted magnetic needle.*

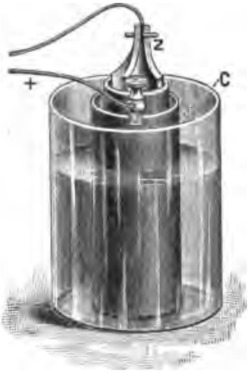

FIG. 87.

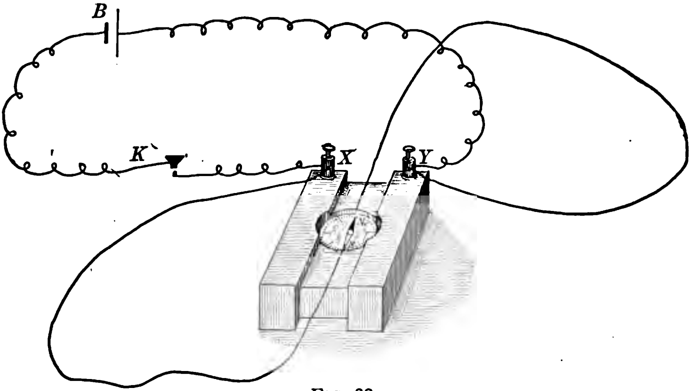

FIG. 88.

Apparatus. — A **voltaic cell**, a **pocket compass**, a **contact key**, and about 3 m. of insulated copper wire. Any type of voltaic cell may be used, but the Daniell cell is best. The Daniell cell (Fig. 87) consists of a copper cylinder, slit down one side, standing in a solution of copper sulphate in a glass jar, and a zinc prism standing in a solution of zinc sulphate, contained in a porous earthenware cylinder placed within the copper one. When the cell is not in use the zinc should be removed, the zinc sulphate poured into a bottle, and the porous earthenware cylinder submerged in a tank of water. This cell does not polarize, but gives a constant current, an essential feature for accurate work. The compass should be mounted in a grooved rectangular wooden block about 10 cm. square by 4 cm. thick (Fig. 88). The block should be provided with two binding posts. The contact key (Fig. 89) is a device for readily opening and closing the circuit. It is not essential, but is very convenient. The insulated wire should be double wound and paraffined; it is known as annunciator wire. The size should be No. 24 American Gauge. This will be found flexible and easy to manipulate.

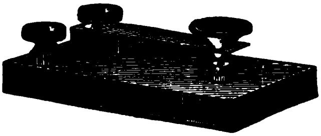

FIG. 89.

Directions. — Set up the battery and connect its poles to the binding posts on the compass block with the contact key in circuit, using about 1.5 m. of wire for the purpose (Fig. 88). Connect the two binding posts on the compass block with the remaining wire. In making these connections, the insulation should be carefully removed from the ends of the wire. Why? The connections must be snug and all contacts clean. *Avoid kinking the wire.* Turn the compass block so that the *N*-end of the needle reads zero. Now stretch a part of the wire that connects the posts on the block across the compass, parallel with the needle, and so that the current in the wire is from *N* to *S*, and note the effect on the needle when the key is depressed. Turn the wire about so that the current flows from *S* to *N*, and record the effect. Repeat these two tests with the wire beneath the needle. Now wind the wire once around the block, the plane of the loop passing through the needle, and the current flowing over the needle from *N* to *S*. Record the magnitude of the effect. Try, successively, twice around, three times around, and four times around. Now wind all the wire neatly on the block, leaving uncovered a small space of about half a centimeter over the needle. The windings should be evenly distributed on the two sides. Record the effect on closing the circuit.

Remarks. — Care must be taken to keep the zero of the compass scale under the north pole of the needle. No deflection can ever be as great as 90°. Give a rule that will express the direction of the needle's deflection. Show how to apply this rule in ascertaining the direction of an electric current in a conductor. On what does the amount of the deflection depend? Does it depend on the direction of the current? On the nearness of the conductor to the needle? On the number of times the current encircles the needle? Of what value are these facts?

Form of Record. — Results may be tabulated as follows: —

POSITION OF CONDUCTING WIRE	DIRECTION OF CURRENT	EFFECT ON NEEDLE
One wire above needle	*N* to *S*	*N*-end deflected ——, —— deg.
One wire above needle	*S* to *N*	*N*-end deflected ——, —— deg.
One wire below needle	*N* to *S*	*N*-end deflected ——, —— deg.
One wire below needle	*S* to *N*	*N*-end deflected ——, —— deg.
One wire around needle	*N* to *S* over	*N*-end deflected ——, —— deg.
Twice around needle	*N* to *S* over	*N*-end deflected ——, —— deg.
Three times around n'le	*N* to *S* over	*N*-end deflected ——, —— deg.
Four times around n'le	*N* to *S* over	*N*-end deflected ——, —— deg.
Many times around n'le	*N* to *S* over	*N*-end deflected ——, —— deg.

110. Problem. — *To construct an electromotive series.*

Apparatus. — **Rectangular strips** of zinc, lead, iron, copper, tin, aluminium, and carbon; a **glass tumbler**, two thirds full of an electrolytic substance, the **galvanoscope** of Art. 109, connecting wires, and a stand for supporting the metal strips. The metal strips are about 3 cm. wide by 10 cm. long, with a hole 3 mm. in diameter through one end, and may be cut from sheets of these metals with tinner's snips. The electrolyte may be zinc sulphate, sulphuric acid, hydrochloric acid, common salt, caustic potash, or copper sulphate. A convenient stand for supporting the electrodes is shown in Fig. 90. Small nuts fasten the electrodes to the metal rods. These rods slide freely through the wooden upright, and are slotted at one end for receiving the connecting wires.

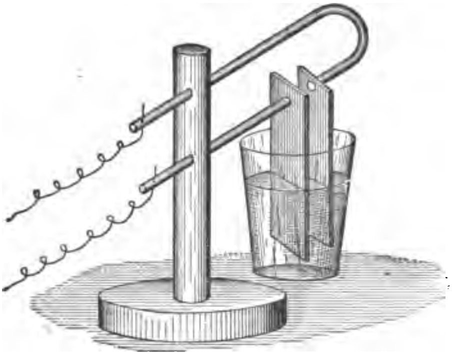

FIG. 90.

Directions. — Attach two electrodes, as copper and zinc, to the rods on the support, and connect these rods to the galvanometer so that the copper electrode is joined to the wire leading *over* the compass needle. Now set the galvanometer so that the reading is zero, then, watching the needle, lift the support and place the electrodes in the liquid. Note carefully the direction of deflection of the north end of the needle. Hold the right hand over the needle with the thumb extended in the direction of the deflection, then the fingers will point out the direction of the current in the conductor over the needle (109). Observe the connections and ascertain from which electrode the current flowed, and mark that electrode +. Repeat the test to verify the conclusion.

Now remove the zinc electrode and substitute another one of the set and repeat the tests. Continue in this manner till every substance has been compared with copper.

Remove the copper and replace it with iron, putting the copper to one side. Proceed as before till every plate has been compared with iron. Then remove the iron, putting some other metal in its place, and proceed as before.

The first member of the required series will be the one that was plus to all the others, the second one will be the electrode that was negative once, the third the one that was negative twice, and so on.

Remarks. — An electromotive series is a table of electrodes such that if any two are selected to make a voltaic cell the electric current through the wire will always flow from the one standing first in the series. The number of combinations possible of a set of the electrodes is one half of the product of the number of electrodes by one less than that number. For example, six metals will give $(6 \times 5) \div 2 = 15$ pairs. The order of the series is likely to vary with the electrolyte. An electrolyte that has been in use for a short time becomes contaminated by the solution of the metals used. The presence of these substances will often change the direction of the electromotive force. This is true in the case of aluminium, iron, lead, and tin when using dilute sulphuric acid as an electrolyte, since any zinc sulphate in the solution is likely to affect the order of these metals in the series. Hence, when comparing these, it is best to try fresh liquid for each pair. The galvanometer described for use in this problem is not a very sensitive one, and in some cases very close observation may be necessary to detect the deflection.

100 c.m 30 Cu wire. L.R – external,

35 –

30 –

Form of Record. — The observations may be recorded after the following plan: —

	ELECTRO-LYTE	ELECTRODES	CONNECTIONS	BEHAVIOR OF GAL.	INFERENCE
1	H_2SO_4	Cu. and Zn.	Cu. to wire over needle *S* to *N*	*N*-end to *W*	Cur. flows from Cu. ∴ Cu. is +
2	"	Cu. and Fe.	Cu. to wire over needle *S* to *N*	*N*-end to *W*	Cur. flows from Cu. ∴ Cu. is +
3	"	Cu. and Al.	Cu. to wire over needle *S* to *N*	*N*-end to *W*	Cur. flows from Cu. ∴ Cu. is +
4	"	Cu. and Pb.	Cu. to wire over needle *S* to *N*	*N*-end to *W*	Cur. flows from Cu. ∴ Cu. is +
5	"	Cu. and Sn.	Cu. to wire over needle *S* to *N*	*N*-end to *W*	Cur. flows from Cu. ∴ Cu. is +
6	"	Cu. and C.	Cu. to wire over needle *S* to *N*	*N*-end to *E*	Cur. flows from C. ∴ Cu. is —
7	"	Fe. and C.	Fe. to wire over needle *S* to *N*	*N*-end to *E*	Cur. flows from C. ∴ Fe. is —
		etc.	etc.	etc.	etc.

Electromotive series: C., Cu., etc., ——.

40 –

30 – wire →

III. Electrical Measurements.

111. The **standard cell** legalized by act of Congress is the Clarke cell, which when constructed according to the specifications of the act has an electromotive force of 1.434 volts at 15° C. For the purposes of this book the Daniell cell (109) will be used as the standard. To secure constant action, it should be placed on short circuit for at least 15 minutes before using, that is, the poles should be connected by a short piece of copper wire. The E.M.F. will be more constant if the solutions are saturated, and it will be slightly higher if the zinc is kept well amalgamated (154). Under these conditions the E.M.F. is about 1.1 volts.

112. The **tangent galvanometer** should be constructed with two or three coils so that it may be adapted to different resistances in the circuit in which it is used. This instrument may have the form shown in Fig. 91, for the construction of which see Art. 140, or what is fully as good and much less expensive, it may be made as shown in Fig. 92, for the construction of which see Art. 141. When in use *the coil must be parallel to the needle*, and hence its plane must be in the magnetic meridian. A very large deflection may often be advantageously cut down by supporting above the needle and in the magnetic meridian a bar magnet with its south pole to the north. On the other hand, a small deflection may be increased by placing this magnet with its north pole to the north. In the first case, the earth's field is strengthened and hence a greater magnetic force on the part of the coil will be needed to overcome it and deflect the needle. In the second case, the reverse is true.

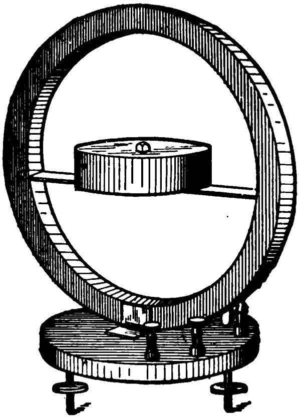

FIG. 91.

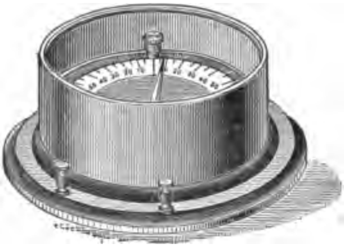

FIG. 92.

113. The **reflecting galvanometer** is, usually, either of the Thomson or the d'Arsonval type. There are different methods of using it: (*a*) By means of a lamp and lens a spot of light is thrown on a graduated scale and this viewed in a subdued light (Fig. 93). (*b*) An inverted scale placed at some distance in front of the mirror is viewed through a small telescope by the aid of light reflected from the mirror. (*c*) A scale at a short distance in front of the mirror is viewed through a small hole just above the scale (Fig. 94). The galvanometer should be one of considerable resistance, that is, its coil should be one of a large number of turns of fine wire.

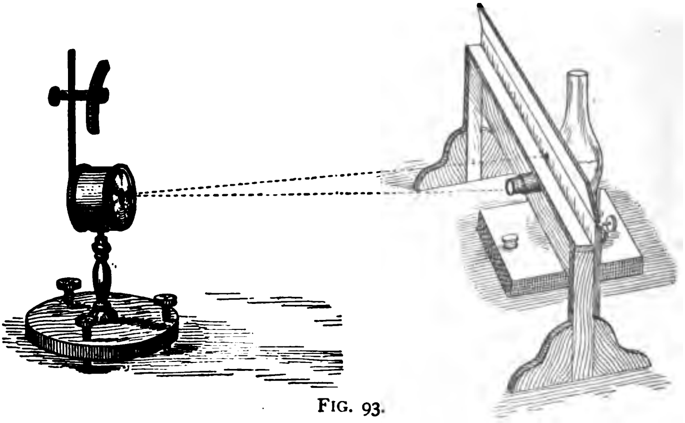

FIG. 93.

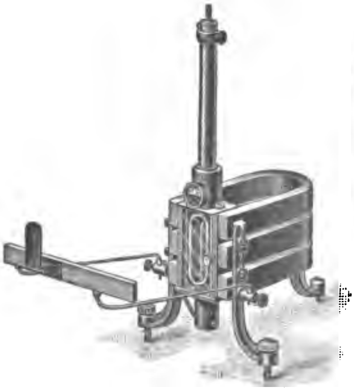

FIG. 94.

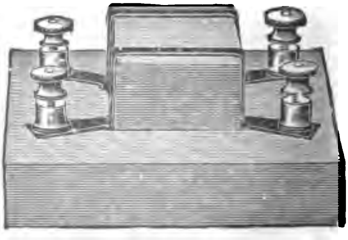

FIG. 95.

114. The **commutator** (Fig. 95) is a device for reversing the current. One of the simplest forms is that known as the "four-hole" commutator. For the manner of construction see Art. 139. Connection must be made to the posts as marked on the base of the instrument. To reverse the current, rotate the top through an angle of 90°.

115. The **contact key** (Fig. 89) should have a very flexible arm, so that light pressure will close the circuit. The contacts should be platinum points, and there should be no rocking motion to the key. In using such a key, the wrist should rest on the table to steady the hand and the key be depressed with an even pressure. Steadiness of contact is very essential, otherwise the resistance of the contact is not constant and the needle of the galvanometer in the circuit will not come to rest.

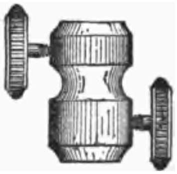
FIG. 96.

116. The **double connector** (Fig. 96) is a simple device for connecting two wires. This is to be used instead of twisting the wires together.

117. The **resistance box** usually consists of a collection of coils of insulated German silver wire, the electrical resistance of each having some known relation to the standard unit of resistance, the **ohm.** By means of movable arms (Fig. 97), or plugs (Fig. 98), as the case may be, any required resistance can be introduced in circuit with the battery. In resistance boxes provided with switches, known as the "dial" type, each

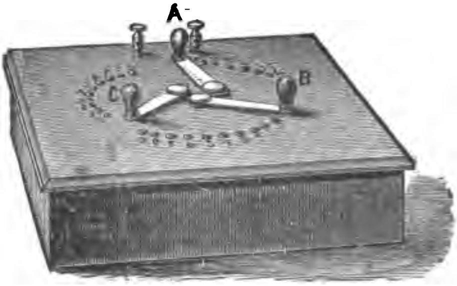

FIG. 97.

switch must rest on a brass nail-head of the arc of nail-heads of which it is the center. The value of the resistance introduced is obtained by adding together the readings adjacent to each switch. In the case of "plug" resistance boxes, the plug is removed to introduce resistance, and the total resistance introduced is the sum of the readings opposite the removed plugs.

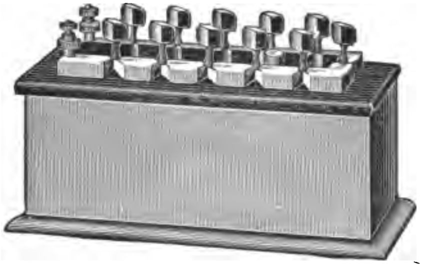

FIG. 98.

118. Problem. — *To measure the resistance of an electrical conductor.*

Apparatus. — A **battery, resistance coils, contact key,** sensitive **galvanometer, Wheatstone bridge, wires** for connections, and **coil of insulated copper wire** whose resistance is to be found. The preferable form of galvanometer is the d'Arsonval, since it is "deadbeat," that is, its index comes to rest without delay. Figure 99 shows the plan of the Wheatstone bridge (142).

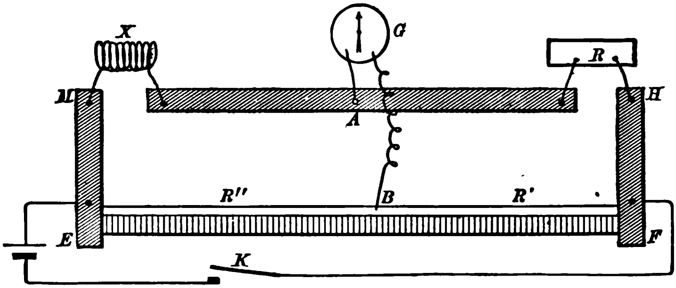

FIG. 99.

Directions. — Connect the apparatus together as shown in Fig. 99. The position of the galvanometer will be determined by the form of instrument used. That shown in Fig. 94 should be placed between the bridge and the edge of the table. In that case the battery wires must be carried along the opposite side of the bridge, keeping the position of the contact key, however, near the left hand of the experimenter. The galvanometer should always be placed so that the slightest motion of the indicator can be seen. If the galvanometer is not provided with a controlling magnet, then the bridge must be placed east and west so that the pointer of the galvanometer when at rest is directed toward the experimenter.

To operate the apparatus, introduce resistance at *R*, then close the battery key and find by trial a position for the sliding key on the bridge wire at which the galvanometer is not affected. The law of the bridge is that when no current flows across it, the products of the resistances of the opposite arms are equal, that is, $R'X = RR''$. Hence, $X = \frac{RR''}{R'}$. R' and R'' as read from the scale are not resistances, but lengths. But the ratio of these lengths is that of their resistances, hence these lengths may be substituted for the resistances in the formula.

The work should be repeated a number of times, changing the conditions each time by introducing different resistances at *R*. The mean of the several values obtained may be considered as the value of *X*.

Remarks. — Be sure that all contacts are clean and snug. Use at least No. 16 copper wire in connecting the resistance coils and conductor to be measured to the bridge and as short pieces as possible. Why? The galvanometer wires must be flexible; copper wire, No. 24, double wound, is recommended. The galvanometer should respond to a change of one millimeter space on the scale. The point on the scale at which the galvanometer reading is zero is quickly found by first finding two positions for *B* on *EF* at which the galvanometer deflections are in opposite directions. Then the place sought is between these. If the galvanometer is "deadbeat," the best way to proceed is to close both battery and galvanometer key, the latter being placed considerably to one side of the middle of the scale; then slide the galvanometer key along the bridge wire till the galvanometer reading changes to zero. Now open both keys, then close them again, the battery key first, to make certain that the balance is perfect. It is best to give *R* a value somewhat near that of *X*, then the balance is secured near the center of the scale, and small errors in the position of the key will have less effect on the accuracy of the results.

Form of Record. — Data may be tabulated as follows: —

R	R″	R′	X
——	——	——	——
——	——	——	——
etc.	etc.	etc.	etc.
		Mean	—— ohms.

119. Problem. — *To find the effect on resistance of connecting two electrical conductors in parallel.*

Apparatus. — Same as in Art. 118.

Directions. — Measure each conductor as in Art. 118. Now connect the two conductors in parallel so that each acts as a shunt to the other, and measure the resistance as before. Compare this value with that obtained by calculation from the law of the divided circuit of two branches, namely, $R = \frac{R_1 R_2}{R_1 + R_2}$, in which R_1 and R_2 are the resistances respectively of the two conductors or branches, and R is the resistance when these conductors are joined in parallel.

Remarks. — Use double connectors for joining the conductors in parallel.

Form of Record. — Tabulate as in Art. 118.

120. Problem. — *To show the effect of temperature on electrical conductivity and determine the temperature coefficient of resistance of an electrical conductor.*

Apparatus. — As in Art. 118, together with a **copper beaker**, and a **pan of water** in which to heat the beaker.

Directions. — Place the coil of wire in the copper beaker, inserting a thermometer and closing the beaker loosely with a plug of cotton wool to cut off air currents. Stand this beaker in a vessel of water. When the temperature of the coil has become stationary, record the temperature and then measure the electrical resistance as already described (118). Raise the temperature of the coil about 10° C. by raising that of the bath. Again measure the resistance and record the temperature. Measure the resistance at temperature intervals of about 10° till 60° C. is reached. Plot a curve, representing the temperatures as abscissæ and the resistances as ordinates. Continue the curve till it cuts the axis of Y and ascertain from the curve the probable point at which the resistance becomes zero. For metals, not alloys, the curve will be nearly a straight line, and will cut the axis of Y at a point very near to the absolute zero of temperature.

The mean temperature coefficient is the mean rate of change in resistance per degree and is found by dividing the total increase in resistance by the total change of temperature, and this quotient by the resistance at the lowest temperature employed.

Form of Record. — The student is to devise one.

121. Problem. — *To measure the resistance of a conductor, as an incandescent electric lamp, by means of a voltmeter and an ammeter.*

Apparatus. — A **voltmeter**, an **ammeter**, and an incandescent **electric lamp**. The voltmeter should have a range sufficiently wide to measure the fall of potential between the terminals of the lamp, and the resistance of the ammeter should be known.

Directions. — Connect the ammeter in circuit with the lamp and source of current and note the current flowing. Then connect the voltmeter as a shunt between one terminal of the lamp and the negative terminal of the ammeter, and record its reading. Then if x is the resistance of the lamp, R that of the ammeter, E the reading of the voltmeter or the fall of potential between the points of contact of the voltmeter, and I the current flowing, we have by Ohm's law $I = \frac{E}{x+R}$; from which $x = \frac{E}{I} - R$.

Remarks. — The above method makes it possible to compare the resistance of a lighted incandescent lamp with that of the same lamp when cold.

Form of Record. — The student should devise one.

122. Problem. — *To measure the electrical resistance of a single cell, and also of two joined* (a) *in parallel and* (b) *in series.*

First Method. **Apparatus.** — A **battery**, a **tangent galvanometer** of known resistance, a **contact key**, a set of **resistance coils**, a **commutator**, and a meter or more of No. 16 copper wire for connections.

Directions. — By Ohm's law $I = \frac{E}{R + r + g}$, in which R, r, and g represent the external resistance, the battery resistance, and the galvanometer resistance respectively. If the external resistance is changed to R', then $I' = \frac{E}{R' + r + g}$. Hence, $\frac{I}{I'} = \frac{R' + r + g}{R + r + g}$. If a and a' are the deflections of the galvanometer under the two resistances, then by the law of the tangent galvanometer $\frac{I}{I'} = \frac{\tan a}{\tan a'}$. Therefore, $\frac{\tan a}{\tan a'} = \frac{R' + r + g}{R + r + g}$, from which r is easily derived.

Place the cell whose resistance is to be measured in circuit with a tangent galvanometer, commutator, contact key, and resistance coils (Fig. 100). Use as little wire as possible in setting up the circuit so that the resistance of the connections may be neglected. Adjust the galvanometer, using the leveling screws for the purpose, till the ends of the galvanometer needle do not differ more than 1° or 2° in their readings for any deflection. This condition must also hold true when the current is reversed. If there is much difference in the readings, it will be necessary to level the instrument while the needle is deflected. Now give R such a value that the deflection, a, of the galvanometer is about 55°. Then change R to R' so that

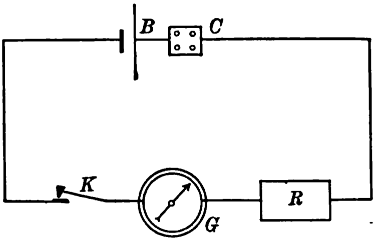

FIG. 100.

a' is about 35°. Both a and a' are the mean of four readings, obtained by reading both ends of the needle and then reversing the current and reading again. This will correct for any curvature in the pointer and also for any small inexactness in placing the needle in the center of the magnetic field of the galvanometer coil. Substitute these values in the formula $\frac{R'+r+g}{R+r+g}=\frac{\tan a}{\tan a'}$, and solve for r. The tangents of the angles are obtained from Table XIX of the Appendix.

In like manner measure the resistance of a second cell of the same kind and size. Now join the two cells in parallel circuit and measure their resistance. Compare this value with that calculated from the value of each cell by applying the law of the divided circuit of two branches, the two cells being considered as these branches. Also join the two cells in series and measure the resistance.

Remarks. — The resistance of a battery is an uncertain quantity, varying with any change in the temperature and with any change in the external resistance in the circuit, or, in general, with the amount of current it is yielding. Hence, successive measurements of the same cell will give varying results, and especially if the battery polarizes readily. The resistance of the circuit outside of the battery must not be very large, otherwise any percentage error in this resistance might absorb the entire resistance of the battery. On the other hand, a large external resistance retards polarization. The reason for choosing 55° and 35° as the deflections is that the tangent of the former is about twice that of the latter, and the needle being equally distant from 45° will be about equally sensitive. The ratio of tan 55° to tan $35\frac{1}{2}$° is almost exactly 2. The formula then reduces to $r=R-2R-g$.

Form of Record. — Results may be tabulated as follows: —

Battery	Mode of Connection	R	Gal. Readings	a	$\tan a$	R'	Gal. Readings	a'	$\tan a'$	g	r
—			—				—				
—	Single	—	—	—	—	—	—	—	—	—	—
			—				—				
—	Single	—	—	—	—	—	—	—	—	—	—
—											
			—				—				
—	Parallel	—	—	—	—	—	—	—	—	—	—
			—				—				
—	Series	—	—	—	—	—	—	—	—	—	—

Second Method. **Apparatus. — Wheatstone bridge, resistance coils, contact key, galvanometer.** — The galvanometer may be of the tangent type (112). If a d'Arsonval or Thomson form be used, it will have to be shunted so that the pointer, or spot of light, will not be thrown off the scale. To shunt a galvanometer is to connect its poles by a copper wire; in this case the wire must be of low resistance. A suitable length of wire must be found by trial.

Directions. — If the battery be placed in one of the arms of a Wheatstone bridge, for example, in place of the X (Fig. 101), when the galvanometer circuit is closed the needle will be deflected. If R, R', and R'' have such values that the closing of the key in the battery circuit has no effect on the magnitude of the galvanometer deflection, then $R'r = RR''$ and $r = \frac{RR''}{R'}$. Hence, set up a circuit as in Art. 118, except that the cell to be measured takes the place of the X, leaving the contact key in the battery circuit. If the galvanometer is of the tangent type, it will be necessary to place it between the bridge and the edge of the table for ease in reading. After adjusting the galvanometer to zero, introduce at R a resistance, the estimated value of r in whole numbers. If the galvanometer deflects beyond 45°, either shunt it, or reduce its deflection by supporting a bar magnet over it, with its north pole over the south pole of the needle. Now find as quickly as possible two places for the galvanometer key, such that when the battery key is closed the galvanometer reading increases in the one instance and decreases in the other. There is a place between these two at which the deflection of the galvanometer does not change on closing the battery key. When this position is found, note the values of R, R', and R'', and calculate r by the formula $r = \frac{RR''}{R'}$.

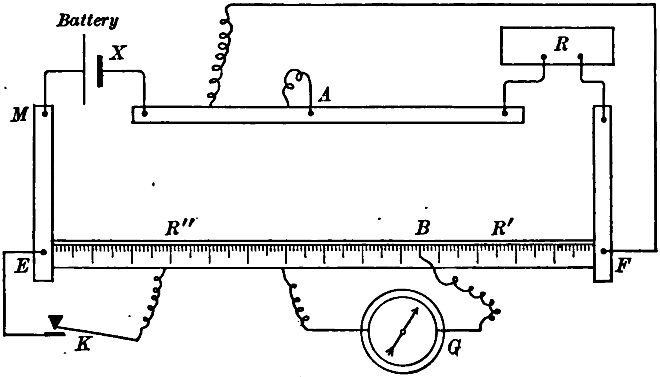

FIG. 101.

Now measure the resistance of a second similar cell, then join the two cells successively in parallel circuit and in series, and measure the resistance, checking the first case by applying the law of the divided circuit of two branches, the two cells being considered as these branches, and the last case by taking the sum of the resistances of the two cells.

Remarks. — If the galvanometer is not "deadbeat," considerable time will be spent in waiting for the needle to come to rest. For cells that polarize the resistance is a growing quantity, and no perfect balancing of the bridge is possible. It follows that the work must be done as quickly as possible, and time will be saved by observing that the indications of the needle are sufficiently marked during the first few positions of the galvanometer key to determine in which direction to move the key, without having to wait for the needle to come to rest. It is advisable to practice the method by measuring the resistance of a non-polarizing cell before attempting one that polarizes.

Form of Record. — It is quite similar to that used in Art. 118.

123. Problem. — *To test a tangent galvanometer by ascertaining how closely it conforms in its readings to the law of tangents.*

Apparatus. — **Resistance coils, Daniell cell, commutator, contact key**, wire of known resistance for connections, or wires of such low resistance that their values may be neglected, and a **tangent galvanometer**.

Directions. — By Ohm's law $\frac{I}{I'} = \frac{R'}{R}$ in which R represents the total resistance of the circuit. By the law of the tangent galvanometer, $\frac{I}{I'} = \frac{\tan a}{\tan a'}$. Hence, $\frac{R'}{R} = \frac{\tan a}{\tan a'}$, and $R \tan a = R' \tan a'$. From this it appears that in the case of a tangent galvanometer $R \tan a$ is a constant quantity, and may be used as a test of such an instrument.

Measure the resistance of the battery, the galvanometer, and the wires to be used in making the circuit. Then join in circuit the battery, galvanometer, commutator, contact key, and resistance coils (Fig. 100). Carefully adjust the galvanometer to position, and introduce sufficient resistance to reduce the deflection to about 5°, the exact value being the mean of four values, the readings of both ends of the needle before and after reversing the current in the circuit. Reduce the resistance sufficiently to give a a value of about 10°, obtained as before, and so on by intervals of about 5° till a reaches a value of about 60°. Multiply the total resistance by the tangent of the corresponding angle of deflection (Table XIX of the Appendix). The degree of agreement among these products will indicate the closeness with which the galvanometer conforms to the tangent law.

A graphic representation of the data is obtained by plotting a curve, using the tangents as abscissæ and the total resistances as ordinates (15). This curve is a hyperbola, and may be used to show what deflection of the galvanometer may be expected for any resistance in the circuit other than that used, as well as to point out any errors made in securing the data. In plotting the curve it is better to consider in the case of the tangents that the unit of value is expressed in terms of the second decimal place. For example, if the tangent should be 0.144, consider it as 14.4 in fixing the scale of values on the X-axis.

Remarks. — It may be necessary in order to secure the larger galvanometer deflections to add another cell. In that case its resistance must be measured, and its value made a part of the total R at that point where it was introduced.

Form of Record. — The following example illustrates the mode of tabulation: —

R	R'	R''	r	
2.0 ohms 1.5 ohms	46.1 cm. 38.9 cm.	53.9 cm. 61.1 cm.	2.34 ohms 2.36 ohms	Mean 2.35 ohms

R_1	r	g	Total R	Gal. Reading	a	tan a	R tan a
ohms	ohms	ohms	ohms				
60	2.35	0.079	62.429	4°.8 – 4°.5 4°.8 – 4°.6	4°.68	0.082	5.119
25	2.35	0.079	27.429	10°.5 – 10°.0 11°.0 – 11°.0	10°.62	0.187	5.129
15	2.35	0.079	17.429	15°.0 – 16°.0 16°.9 – 17°.0	16°.22	0.291	5.072
11	2.35	0.079	13.429	21°.0 – 21°.0 20°.0 – 21°.5	20°.87	0.381	5.116
8	2.35	0.079	10.429	26°.0 – 26°.0 26°.5 – 26°.2	26°.17	0.492	5.131
6.3	2.35	0.079	8.729	30°.5 – 30°.6 30°.0 – 29°.5	30°.15	0.581	5.072
5.0	2.35	0.079	7.429	34°.5 – 35°.0 34°.0 – 35°.0	34°.62	0.690	5.126
3.5	2.35	0.079	5.929	40°.5 – 41°.0 40°.7 – 40°.5	40°.68	0.859	5.093
2.0	2.35	0.079	4.429	49°.5 – 50°.0 50°.0 – 49°.5	49°.75	1.181	5.231
0.5	2.35	0.079	2.929	59°.5 – 60°.5 61°.5 – 61°.0	60°.62	1.777	5.205

124. Problem. — *To calibrate a tangent galvanometer so that it will measure the current in amperes, or milliamperes.*

Method by Ohm's Law. **Apparatus.** — **Daniell cell, commutator, resistance coils, contact key, tangent galvanometer,** and wires for connection.

Directions. — The law of the tangent galvanometer is that the current is proportional to the tangent of the angle of deflection, that is, $I = K \tan a$, in which K is a constant, called the *Reduction Factor*, and its value varies with the construction of the instrument. From this $K = \frac{I}{\tan a}$, and by Ohm's law, $I = \frac{E}{R + r + g}$. Hence, $K = \frac{E}{(R + r + g) \tan a}$, in which E is 1.1 volts, if a Daniell cell is used.

Measure the resistance of the cell and galvanometer, then join in circuit the battery, resistance coils, contact key, commutator, and galvanometer. Carefully adjust the galvanometer, and introduce resistance, R, to make a about 45°. In obtaining a, read both ends of the needle, reverse, and average the four readings. By substituting the values of R, g, a, r, and E in the formula $K = \frac{E}{(R + r + g) \tan a}$, the value of K can be found. If any value of $\tan a$ is multiplied by this value of K, the product will be approximately the current in amperes flowing through the galvanometer when this value of a is given.

To construct a calibration curve, compute the current for the angles 5°, 10°, 15°, etc., to 60°. By moving the decimal point three places to the right, these current values will be expressed in milliamperes. Use the angles as abscissæ and the milliamperes as ordinates, and through the points thus obtained draw a smooth curve. With this calibration curve it will be possible to measure the current passing by finding the point on the curve whose abscissa is the galvanometer reading.

Remarks. — If a student has solved Problem 123, then the data of that problem can be used in finding the value of K.

Form of Record. — See Art. 123.

Method by Copper Voltameter. **Apparatus.** — A **tangent galvanometer** to which a **commutator** is connected in order that the current through the galvanometer can be reversed at pleasure, an **adjustable resistance**, a **key** for closing the circuit, preferably a plug key, two **Daniell cells**, and a **copper voltameter**. The voltameter consists of a glass vessel containing a solution of copper sulphate, and dipping into this solution are two copper plates, serving as anode and cathode respectively, attached by spring clips, so as to be easily removable, to a wooden bar resting across the glass jar. These electrodes are rectangular in shape, each provided with an ear for attaching to the supporting bar. The anode is bent into a U-shape, and the cathode is situated between its arms, so that the copper is deposited on both faces of it. The cathode should be about half an inch from the anode, and have an exposed area of about 50 cm.^{2} per ampere of current used. The copper sulphate solution is prepared as follows: $CuSO_4$, 15 gm.; H_2SO_4, 5 gm.; alcohol, 5 gm.; water, 100 gm.

Directions. — Both plates must be clean, but the cathode must be cleaned with more than usual care. This is done by polishing it with fine sandpaper till all parts are bright and then rinsing in water flowing from the tap. Dry by rinsing in alcohol. Avoid touching with the fingers the parts that are to be immersed in the electrolyte. Weigh accurately to milligrams. Connect the apparatus in series, joining the cathode to the zinc pole of the battery. Adjust the number of cells and the resistance till the deflection of the galvanometer is about 45°. Let the current run for half an hour; read the deflection of the needle every five minutes, before and after reversing the current through the galvanometer, to ascertain whether the current is constant. Adjust the resistance, if found necessary, to maintain a constant deflection. At the end of 30 minutes remove the cathode; rinse it in water, then in alcohol, dry quickly, and weigh as before. Record the weights of the cathode, the deflection of the galvanometer, the time that the circuit was closed. Since $K = \frac{I}{\tan a}$, where a is the average deflection of the galvanometer, and one ampere deposits 0.0003287 gm. of copper in one second, then

$$I = \frac{W \text{(the gain in weight of the cathode)}}{0.0003287 \times t},$$

and $$K = \frac{W}{0.0003287 \times t \times \tan a}.$$

After finding K proceed as in the first method.

Form of Record. — The results may be tabulated as follows: —

t	a			WEIGHT OF CATHODE		CU. DEPOSITED	K
	Direct	Reversed	Mean	Before	After	W	
——	——	——	——	——	——	——	——

125. **Problem.** — *To prove Ohm's law, namely, that the fall of potential in an electrical conductor carrying a current is proportional to the resistance.*

Apparatus. — A **d'Arsonval galvanometer** of considerable resistance, also a **resistance coil** of at least 5000 ohms, two or three **Daniell cells** joined in series, a **contact key**, and a meter of very high resistance wire stretched lengthwise on a meter rod. "Climax" wire, No. 30, will answer the purpose.

Directions. — Connect the battery and contact key to the ends of the wire on the meter rod, and the galvanometer and high resistance to one end of the wire and the slide key (Fig. 102). Close the battery key, and then close the slide key successively at intervals of, say, 10 cm. along the wire and record the galvanometer reading at each place. The difference in potential between the points on the wire tested will be proportional to the current sent through the galvanometer each time. Since the deflections of the galvanometer are small, the current may be considered as proportional to the deflection. The resistance of the wire may be taken as proportional to the length. Hence, if Ohm's law is true, the quotient of the length of the wire in each case by the corresponding galvanometer deflection should be constant. Plot the data, using the wire lengths as abscissæ and the galvanometer readings as ordinates.

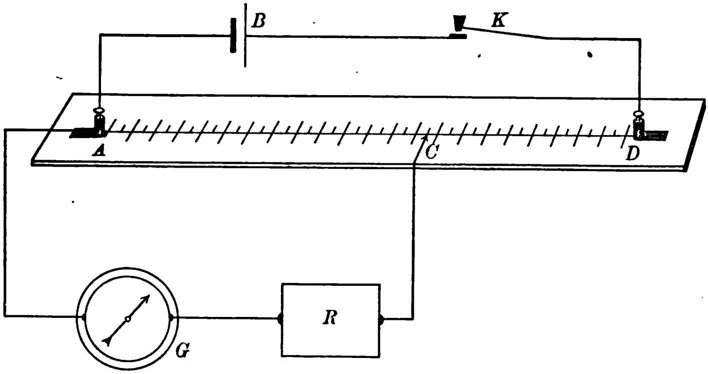

FIG. 102.

Remarks. — It should be observed that the galvanometer having a resistance of 5000 ohms in series with it is practically a voltmeter. A voltmeter may be substituted for the galvanometer and the high resistance coil. It would be better to connect the battery through a commutator, then make each galvanometer reading the mean of two readings, one obtained before reversing the current through the wire and the other after doing so. An efficient high resistance for use in this problem in series with the galvanometer can be made by drawing a pencil line on a strip of ground glass. Contacts can be made to the ends of this line by clamping strips of tin foil to the glass. Copper wires can be easily connected to the tin foil.

Form of Record. — The data may be recorded as follows: —

Readings on Wire		l	Gal. Readings		a	$\frac{l}{a}$
—	—	—	—	—	—	—
—	—	—	—	—	—	—
—	—	—	—	—	—	—
etc.	etc.	etc.	etc.	etc.	etc.	etc.

126. Problem. — *To measure the electromotive force of a battery, and also of two such cells joined* (a) *parallel and* (b) *in series.*

Apparatus. — A **standard cell**, either Daniell or Clark, a **tangent galvanometer**, a **commutator**, a **contact key**, a set of **resistance coils**, and No. 16 copper wire for connections.

Directions. — *Constant Resistance Method.* By Ohm's law $I = \frac{E}{R + r + g}$, and for a second battery $I' = \frac{E'}{R' + r' + g}$. Hence, $\frac{I}{I'} = \frac{R' + r' + g}{R + r + g} \cdot \frac{E}{E'}$. But for a tangent galvanometer, $\frac{I}{I'} = \frac{\tan a}{\tan a'}$. Therefore, $\frac{R' + r' + g}{R + r + g} \cdot \frac{E}{E'} = \frac{\tan a}{\tan a'}$. If $R' + r' + g = R + r + g$, then $\frac{E}{E'} = \frac{\tan a}{\tan a'}$, that is, the E.M.F. of two cells are to each other as the tangents of the angles of deflection when the resistances of the two circuits are equal. To apply this principle to the problem, *first*, measure the resistance of the standard cell, and, *sec-*

ondly, that of the cell whose E.M.F. is sought. Then join the standard cell in circuit with the galvanometer, resistance coils, commutator, and contact key, using as little wire as possible, so that the resistance of the connections may be neglected in comparison with the total resistance of the circuit (Fig. 100). Introduce sufficient resistance to give the galvanometer a deflection of between 25° and 55°. The galvanometer should be very carefully adjusted, and the angle a should be the mean of the four readings obtained by reading both ends of the needle and then reversing the current and reading again. Now substitute the battery whose E.M.F. is required, adjusting the resistance so that the total is the same as when the standard cell was in circuit. This will be the case if R' is made equal to $R + r - r'$. Now obtain a'. Then $\frac{E}{E'} = \frac{\tan a}{\tan a'}$, and $E' = E \cdot \frac{\tan a'}{\tan a}$, in which $E = 1.1$ volts for the Daniell cell.

In like manner measure a second cell of the same kind. Then join them successively in parallel circuit and in series and measure the E.M.F.

Remarks. — By using a galvanometer whose resistance is several hundred ohms, the resistance coils may be omitted from the circuit and the resistances of the batteries, being relatively small, may be neglected. The method is then reduced to that of taking the reading a, when the Daniell battery is connected to the galvanometer, and a' when the one under investigation is joined. Then $E' = E \cdot \frac{\tan a'}{\tan a}$. Such a galvanometer could be easily calibrated to read in volts. A voltmeter is but a galvanometer of high resistance, so high that the battery resistance and connecting wires sink into insignificance in comparison therewith.

Form of Record. — The results may be tabulated as follows: —

R	R'	R''	r	R	R'	R''	r'
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
			—				—

Battery	Connect'n	R	r	g	Total R	a	R'	r'	a'	E	E'
—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—

Reduced Deflection Method. By Ohm's law $I = \frac{E}{R+r+g}$, and if R is increased by R', so as to reduce the galvanometer deflection by, say, 10°, then $I' = \frac{E}{R+R'+r+g}$. For a second cell, giving the same deflection, $I = \frac{E'}{R''+r'+g}$, and when the deflection is reduced as before by introducing more resistance, $I' = \frac{E'}{R''+R'''+r'+g}$. Hence,

$$\frac{E}{R+r+g} = \frac{E'}{R''+r'+g},$$

and

$$\frac{E}{R+R'+r+g} = \frac{E'}{R''+R'''+r'+g}.$$

By inverting these equations, and subtracting member from member, we have $\frac{R'}{E} = \frac{R'''}{E'}$, that is, the E.M.F. of two cells are to each other as the resistances which produce equal diminutions of current strength. Hence, connect in circuit the Daniell cell, the galvanometer, the resistance coils, and the contact key (Fig. 103). Introduce sufficient resistance to make the galvanometer deflect about 45°, then increase the resistance to make the galvanometer deflect to, say, 35°; the difference between these two resistances will be R'. Replace the Daniell cell by one whose E.M.F. is to be determined, and introduce sufficient resistance to obtain the same deflection as when the Daniell cell was used. Then add resistance till exactly the same change in a is produced as before. The difference between these two resistances will be R'''. Hence, $E' = \frac{R'''}{R'} \cdot E$, in which $E = 1.1$ volts.

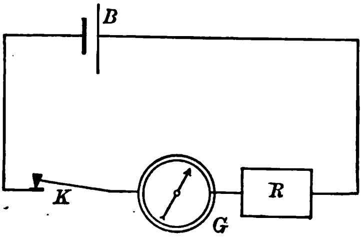

FIG. 103.

In like manner measure two such cells in parallel and also in series.

Remarks. — To prevent polarization, select a galvanometer of high resistance. The d'Arsonval galvanometer will yield excellent results by introducing resistance to keep the deflections small and substituting divisions of the scale for degrees.

Form of Record. — Enter the results as follows: —

BATTERY	CONNECTIONS	1ST R	2D R	DIF. OR R'	3D R	4TH R	DIF. or R'''	E	E'
——	——	——	——	——	——	——	——	——	——
——	——	——	——	——	——	——	——	——	——
etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.

Potentiometer Method. **Apparatus.** — A meter of high resistance wire stretched along a meter rod for a scale (see Art. 125), two **Daniell cells**, or a **storage cell**, a **d'Arsonval galvanometer**, a **coil** of at least 5000 ohms resistance, a **contact key**, a **standard cell**, either Daniell or Clarke, and the batteries to be measured.

Directions. — Connect up a circuit as shown in Fig. 104. When the key, *K*, is closed a current will flow from *A* to *C* on account of a difference of potential between these points, *A* being at a higher potential than *C*. If the positive pole of *B*, the battery to be tested, be connected by a slide key to a point on *AC* as *E*, then the pole of this cell becomes of the potential of that point. Since the potential of *AC* drops to practically zero at *C*, there is a point *F* where the potential is that of the negative pole of *B*, and no current can flow through the circuit *BGR*. Hence, set *E* at some point, as 10, on the scale, slide *F* along the wire till *G* gives no deflection, and record the reading. The difference will be a quantity depending on the difference of potential between the poles of the battery. To correct for

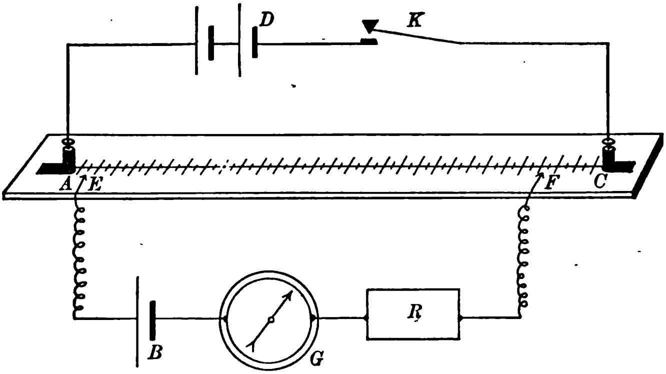

FIG. 104.

inequalities in the wire *AC*, repeat the work with *E* set at different places along *AC*, and obtain the mean of the differences. Now replace the battery, *B*, by the standard cell (Daniell or Clarke) and proceed as before. The ratio of these differences will be the E.M.F. of the battery.

In like manner measure two such cells joined, *first*, in parallel and, *secondly*, in series.

Remarks. — In applying this method, it is necessary that the E.M.F. of *D* be greater than that of the cell to be measured. When the two cells are joined in series, it may be necessary to add another cell to *D*. If storage cells are used, *K* should be closed only during the making of the test, and the wire, *AC*, must be of sufficiently high resistance to cut down the current to a point which will not perceptibly heat the wire.

Form of Record. — Enter the results as follows: —

CELL	CONNECTION	READING AT *E*	READING AT *F*	DIFFERENCE	MEAN	E.M.F.
—	—	— — —	— — —	— — —	—	—
—	—	— — —	— — —	— — —	—	—
etc.	etc.	etc.	etc.	etc.	etc.	etc.

IV. Attraction and Repulsion of Currents.

127. Problem. — *To ascertain the effect that one electric current has upon another.*

Apparatus. — Construct a rectangular frame,[*] 25 cm. square, out of insulated copper wire, No. 20, by winding about four layers around the edge of a square board. Slip the wire off the board, and tie the parts together in a number of places with fine cord. Bend one end of the wire into a hook form (Fig. 105), the other end to be left straight, extending out from the other side. Bend a narrow strip of brass into a J-form, giving the hooked part a spoon shape, so that it will hold a small globule of mercury. Support the strip in a burette holder, and hang the rectangle from it with the straight end dipping into a vessel of mercury. Adjust the shape of the hooked part of the wire so that the upper and the lower sides of the rectangle are horizontal, the frame keeping any position given it, and turning on application of the slightest force. The supporting point of the frame should be conical, good contact being secured by a globule of mercury.

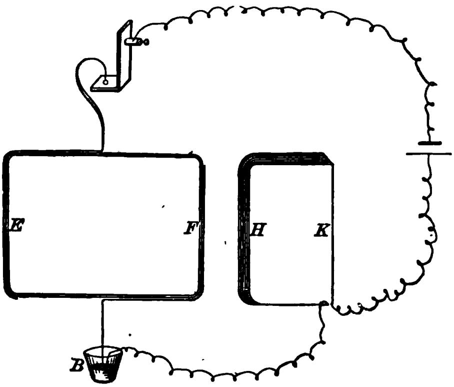

FIG. 105.

Construct a second rectangular helix by winding four or five layers of insulated wire around the edge of a board, *HK*, 20 cm. by 10 cm. For convenience in handling, this helix should be fastened to the board.

Construct a helix of the form shown in Fig. 106 by winding some insulated wire

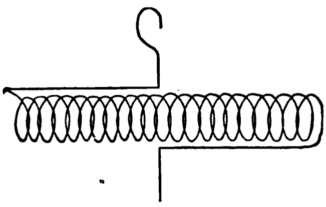

FIG. 106.

into a close spiral, arranging it for suspension in the same manner as in the case of the wire rectangle. The helix or solenoid should have a diameter of about 4 cm. and a length of 15 cm.

Directions. — Place the wire rectangle in circuit with a battery of two or three cells, by connecting one pole to the strip, and the other to the vessel of mercury. If the plane of the rectangle be situated north and south, the rectangle, if carefully mounted, will turn till its plane lies east and west. Reverse the current and record the effect. Trace the current through the rectangle and record its direction.

Place in the same battery circuit the coil *HK*, and support it so that the current through it is parallel to that through *F* and near to it. Record the effect when the direction of the currents through *F* and *H* is down. When the direction in both is up. When down in one and up in the other. Frame a law expressing the results.

Hold *HK* within *EF* so that the edge *H* makes an angle with the lower side of *EF*. Frame a law expressing the results.

Remove *HK* from the circuit, and double part of the wire on itself, as shown in Fig. 107, and hold it parallel to one of the vertical sides of the wire rectangle. Compare the effect with that of a wire not doubled on itself.

Substitute for the simple loop a spiral with the return wire passing in a straight line through it (Fig. 107). If the effect is the same as that of a simple loop, then a sinuous current must equal the straight current.

Place a bar magnet beneath the suspended rectangle and note the effect. Turn the magnet end for end, and again note the effect.

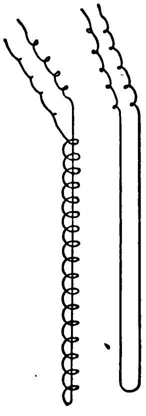

FIG. 107.

Place the rectangle, *HK*, in circuit with the solenoid. Hold one edge of *HK* above the solenoid and parallel to its axis. Compare the direction of the current in *H* with that in the solenoid, on the solenoid's coming to rest. What law is illustrated?

Remarks. — The magnetic properties of a solenoid or helix are very instructively exhibited by obtaining its magnetic field. This may be done by placing the solenoid in a rectangular opening cut in a sheet of cardboard, with the axis in the plane of the cardboard. If, in cutting the rectangle, one end is not cut across, and the strip is passed through the axis of the helix, it would be better. Then sift iron filings around the coil, while a current is passing through it; a field resembling that given by a magnet will be obtained. Now fill the opening through the coil below the cardboard with iron wires, cut the length of the coil, and repeat the experiment. Compare the lines of magnetic force with those obtained before using the wire core.

Form of Record. — Record briefly and clearly the result of each experiment, and point out in the discussion the laws and truths established.

V. Current Induction.

128. Problem. — *To investigate the laws of magneto-electric induction.*

Apparatus. — A **sensitive galvanometer**, a **hollow spool of insulated wire** (Fig. 108), a strong **bar magnet**, and a **battery cell.**

Directions. — Connect the hollow helix by long wires to the galvanometer. Thrust into the helix the N-seeking pole of the bar magnet. Record the direction of the deflection of the needle. Place the battery cell in circuit with the helix and galvanometer so as to deflect the needle in the same direction as when the N-seeking pole of the magnet was inserted in the helix. Trace the direction of the current through the helix. Compare the direction of the induced current with that of the hands of a watch. Remove the battery, and ascertain if there is any current when the magnet is stationary within the helix. Study the effects attending a slow introduction of the magnet. Try the S-seeking pole. Find the effect of suddenly removing the magnet. Try a stronger magnet made by placing two together with their like poles adjacent. Express as laws the conclusions reached.

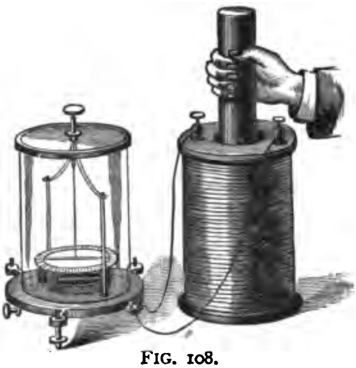
FIG. 108.

Remarks. — If the direction of the winding of the helix cannot be determined by an inspection of the spool, put it in circuit with the battery and with a compass needle ascertain the polarity of the coil. The connections must be made exactly as was done in the experiment. By grasping the coil with the right hand so that the extended thumb points toward the N-seeking pole, the fingers will extend around the coil in the direction in which the battery current is flowing, and hence in the direction of the induced current when the N-seeking pole of the bar magnet was inserted.

129. Problem. — *To investigate the laws of current induction.*

Apparatus.—Two **helices** of wire, one of which can be inserted in the opening in the other (Fig. 109), a **battery cell,** and a **sensitive galvanometer.**

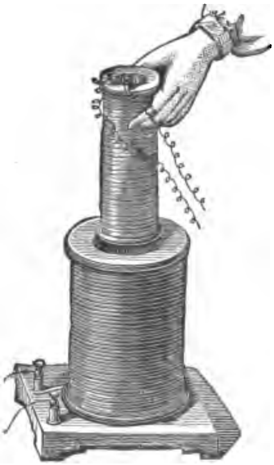

FIG. 109.

Directions.—Connect the smaller helix in circuit with the battery cell. Connect the larger helix to the galvanometer. Insert the smaller helix quickly within the larger one, and note the effect on the galvanometer. Find, as in Art. 128, the direction of the current in the larger helix. Ascertain the effect of removing the helix. Place the smaller helix within the larger one, and try the effect of opening and closing the circuit. Insert a set of resistance coils in the battery circuit, and ascertain if there is any induction produced on weakening or strengthening the battery current by changing the resistance. Within the smaller helix place a bundle of fine iron wire and repeat all the tests, comparing the results with those previously obtained. What laws express the result of the experiments? In the discussion, compare these laws with those of magneto-electric induction, and point out their similarity.

Remarks. — In changing the resistance of the circuit so as to change the strength of the current, it must not be done by interrupting the current. The resistance must be arranged with a sliding contact so that there will be no interruption. Why?

APPENDIX A.

DIRECTIONS FOR MAKING APPARATUS.

130. To etch a Scale on Glass. — Thoroughly clean the piece of glass, and coat it evenly with a thin film of beeswax or paraffin. Fasten the glass on the table, and at a distance of about 50 cm. fasten a steel scale. In a wooden bar, about 52 cm. long, 2 cm. wide, and 1 cm. thick, insert at each end, and perpendicular to it, a stout needle point 2 cm. long (Fig. 110). Place one of the needle points of the bar on the first division of the scale, and with the other point draw a line in the wax, cutting entirely through it. Then place a needle point on the second division of the scale, and so on till a scale of sufficient length has been obtained. Every fifth division should be made a little longer than the others, and every tenth division should be longer still. With a sharp-pointed instrument add the proper figures.

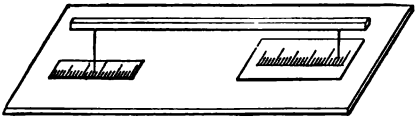

FIG. 110.

Make a rectangular dish out of a strip of sheet lead by turning up the edges. Place in the dish a spoonful of powdered fluor spar thoroughly moistened with sulphuric acid. Cover the dish with the glass plate, scale side down. Warm the vessel slightly over a lamp, being careful not to melt the wax, and set aside for a few hours. Remove the wax with benzine. A scale will be found to be etched in the glass.

131. To etch a Scale on Brass. — Proceed as in Art. 130, except that the etching is to be done by placing on the surface a piece of blotting paper wet with nitric acid.

132. To make a Mirror Scale. — Fasten the piece of mirror plate on the table, silvered side up. Then, as in Art. 130, cut the scale in the amalgam. Cover the surface with red paper. The scale might be etched on the glass face (130).

133. To make a Scale on Paper. — Substitute a pen or pencil for one of the sharp points of the wooden bar of Art. 130, and then proceed as directed in that article.

134. To draw on Glass with Ink. — Clean the glass plate, then lick it with the tongue. When dry, a pen will write on the glass surface almost as readily as on paper.

135. A simple Jolly Balance. — A serviceable one may be made as follows: Provide a stout supporting rod of wood set into a circular wooden block for a base. Suspend a spiral spring from an arm projecting from the top. Two or three of these springs should be provided, varying in stiffness. They can be made by winding spring brass wire closely around a cylinder 1 cm. in diameter. Wires varying from No. 20 to No. 30 may be used, according to the sensitiveness required. To the lower end of the spring attach two small scale pans, one below the other, and about 15 cm. apart. Fasten to the supporting rod back of the spring a long millimeter scale cut on mirror plate (132). Just above the upper scale pan fasten to the wire a small white bead to serve as an index. The scale reading is where the line joining the index with its reflection intersects the scale.

The balance may be made to give results in grams by finding the elongation that a gram weight will produce, and hence the value of a scale division. This value, multiplied by the elongation in any case, will be the weight of the body producing the elongation.

136. A Magnetoscope. — Magnetize a small cylinder of hard steel, about 15 mm. by 3 mm. in diameter, and paint the south end red. Suspend this by a silk fiber in a glass cylinder, about 20 cm. high and 3 cm. in diameter (Fig. 84). A brass screw through a cork fitted to the cylinder will serve to support the fiber. Support the magnet horizontally in a stirrup made of thin brass foil. To prevent pendular swinging, connect a second fiber to the stirrup, and attach it to a lead disk resting on the bottom of the cylinder. Adjust the lengths of the fibers to bring the magnet near the center of the cylinder.

137. An Electroscope. — Select a Florence flask of about a liter capacity; thoroughly clean and dry it; fit to it a paraffined cork or rubber stopper through which passes a slender brass rod, terminating at its outer end in a brass ball about 15 mm. in diameter. File the end of the rod within the flask to a wedge form, and cement to each of these beveled faces a strip of aluminium foil, 3 mm. wide and 5 cm. long. These strips should be adjusted to hang side by side at the middle of the flask.

138. A Two-hole Commutator. — Bore two holes, 1 cm. deep and 1 cm. apart, in a block of wood. Fill these nearly full of mercury, and connect each by a strip of sheet copper to a binding post screwed into the block. The galvanometer is to be connected to the binding posts, the battery to the mercury cups by dipping the ends of the wires into them. To reverse the current, it will be necessary simply to cross the battery wires.

139. A Four-hole Commutator. — On one face of a wooden block, 8 cm. square and 2 cm. thick, lay off a square 2.5 cm. on a side. At each corner of this square bore a hole 1 cm. in diameter and 1 cm. deep. At each corner of the block insert a binding post, and connect the foot of each post to the adjacent hole by a copper strip, bending the end of the strip at right angles so that it will reach to the bottom of the hole (Fig. 95). Prepare a second block of wood 2.5 cm. square and 2 cm. thick, and bend over it two copper strips, 2.5 cm. apart, the ends projecting 8 mm. beyond the face. Fill the holes nearly full of mercury and place this second block on the first with these projecting copper strips dipping into the mercury cups. Now if two diagonally opposite posts are connected to the battery and the other two to the circuit, then by turning the movable block through 90° the current through the circuit will be reversed.

140. Tangent Galvanometer, Ring Form. — Cut out of well-seasoned wood a ring 30 cm. in diameter, and having a cross section of 3.5 cm. square. Mount this on a circular wooden base, provided with leveling screws. In the outer edge of the ring cut a rectangular groove 2 cm. wide and 2 cm. deep. In this groove wind two turns of heavy copper wire, the ends to be connected to binding posts on the base. This will give a coil of practically no resistance. Now wind in the same groove, using insulated copper wire No. 26, three other coils of 10, 30, and 90 turns, respectively, the ends of each to be connected to its own pair of binding posts. Fasten across the ring a wooden or brass bar, supporting a wooden box 12 cm. in diameter and 5 cm. deep. Place on the bottom of this box a circular piece of mirror plate, to which is cemented a paper ring graduated to degrees. For a needle, magnetize a piece of watch spring 15 mm. long, inserting it edgewise into a disk of elder pith (Fig. 111). Cement to the pith disk, and at right angles to the magnet, a pointer or index of aluminium wire, painted black. Suspend the needle by a fine silk fiber from a metal button on the glass top of the protecting box. A small hole is drilled through the glass cover, and the short stem of the metal button reaches through it. A fine hole, drilled axially through this stem, makes it possible to thread the suspending fiber through it and fasten it to the top of the button. It will now be easy to adjust the distance of the needle from the scale, and to turn the needle about till the pointer is on the zero. The needle should hang at the center of the wooden ring, and should swing as close to the scale as possible without touching. To dampen its vibrations it is advisable to cement a thin rectangular strip of mica or aluminium foil, 3 cm. long by 1 cm. wide, to the pith disk. The suspending fiber may then be attached to the center of the top edge of this vane or damper. When the instrument is leveled, the two ends of the pointer should give the same readings, and reversing the current should not alter the amount of the deflection; if it does, the pointer is either not set at right angles to the magnet, or the magnet is not in the center of the ring. To eliminate small errors of this kind, read both ends of the pointer; reverse the current, and again read both ends, and use the mean of these four readings.

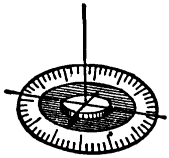
FIG. 111.

141. Tangent Galvanometer, Box Form. — To a circular box, made of wood or hard rubber, fit a glass cover, needle, and scale as in Art. 140. The box should be about 12 cm. in diameter and 10 cm. deep, and the bottom 2.5 cm. thick, with a rectangular hole, 3.5 cm. by 2.5 cm., cut centrally through it. Wind on a wood or hard rubber block, 3.5 cm. by 2 cm. square, a flat coil of several layers of No. 24 insulated copper wire. This coil should be evenly wound, and should be 3 cm. long when completed. Fasten the box with brass screws to a wooden or rubber base, 20 cm. long by 12 cm. wide. Fit the coil in the aperture at the bottom of the box, carry the ends of the wires through the base, and connect to binding posts on the base. The axis of the coil should be crosswise of the base, and the zero diameter of the scale should be lengthwise of the base. The pith disk is omitted from the needle, and both the magnet and the pointer are cemented to the mica damper. The needle should be adjusted to swing about 1 cm. above the coil, and the scale should be close to the needle. Two or more coils of different resistances may be wound on the same block. The base should be fitted with leveling screws. This instrument is much more sensitive than the old tangent type, follows the tangent law just as closely for deflections between 0° and 60°, and can be made for one third the cost.

142. The Wheatstone Slide Bridge. — On a varnished board, 1.1 m. long and 15 cm. wide, fasten with brass screws strips of sheet copper 2.5 cm. wide, in the manner shown in Fig. 99, the pieces *ME* and *HF* being each 10 cm. long and *A* 90 cm. long. The distance between *E* and *F* is exactly 1 m. Solder a stretched, naked German silver wire, No. 26, between *E* and *F*, parallel to and within 5 mm. of the meter rod, *B*. Solder binding posts to the copper strips, so that connections may be made as shown in the figure. The sliding contact for the galvanometer is a wooden block 2 cm. square and 7 cm. long, lengthwise of the top of which, and bent at right angles over one end, is a strip of very flexible spring brass 1 cm. wide. This spring is fastened to the block by two screws near one end, and the adjustment is such that as the block slides along by the edge of the meter rod a gentle pressure on the spring brings the free end that is bent over the block in contact with the wire. A binding post is soldered to this spring at a point between the two screws.

143. To make a Porte Lumière.— Procure a piece of pine board as long as the width of the window, and about 60 cm. wide. In it cut a round hole 12 cm. in diameter, the center of which should be the center of the line parallel to the edge of the board, and 20 cm. from the bottom. On one face of the board, on opposite sides of the center of the hole, *T* (Figs. 112, 113), and having their inner faces 15 cm. apart, fasten two wooden arms, *DE*, 32 cm. long, supported by iron braces, *C*. Pivot between the outer ends of *DE* a rectangular board, *FH*, 2 cm. thick and 15 cm. square, making the distance, *DE*, 31 cm. Cut a half-round piece, *K*, 5 cm. thick, with a radius of 7 cm., and fasten it by slender screws to the rectangular piece, *LM*, 25 cm. by 15 cm. *QN* is a hardwood cylinder, 2.5 cm. in diameter; it may be cut from an old broom handle. In one end, as shown at *O*, cut a slot so that it will clasp the piece, *K*, to which it is to be secured by a bolt, around which it is free to turn, with slight friction. *S* is a triangular block, the shape of which is determined by the method illustrated in Fig. 114. On a large sheet of paper

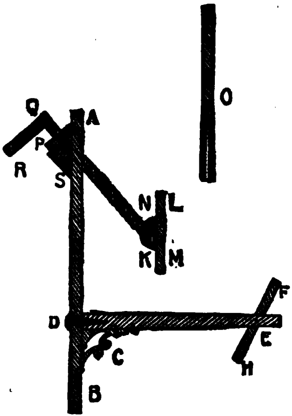

FIG. 112.

lay off the lines *DE*, 31 cm., and *DA*, 40 cm., forming a right angle at *D*, a point corresponding to the center of the hole, *T* (Fig. 113). At *E*, with a protractor, make the angle *DEQ* equal to the latitude of the place. Where *EQ* cuts *DA* is the center of the hole to be bored for *QN*. The upper angle of the piece, *S*, is evidently equal to the angle *DEQ*. Cut this piece, *S*, from a block of wood, and fasten it to *AD*, then bore the hole for *QN* perpendicular to the upper face of *S*. *P* is a wooden washer, pinned to *QN*, to serve as a shoulder, and is so placed that *QN* extends through the board far enough to bring the lower edge of *LM* a few centimeters above the top of the hole, *T*. *R* is a pin at right angles to *QN*, for convenience in turning it. On the front of both *LM* and *FH* fasten a good plane mirror, completing the apparatus.

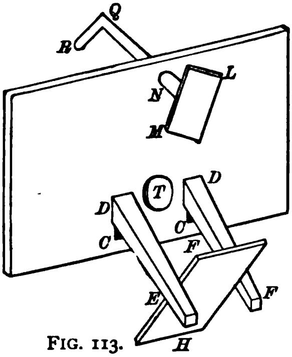

FIG. 113.

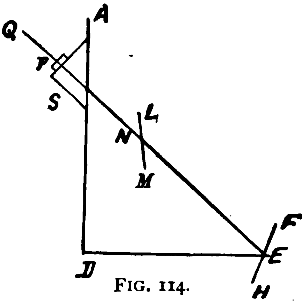

FIG. 114.

Place the porte lumière in a south window, with the board in a vertical position. Determine by trial the inclination of the mirrors, so that sunlight incident on *LM* is reflected to *FH*, and from it in a horizontal line through *T*. Now, by turning *QN* on its axis, the beam of light can be kept in one place, notwithstanding the continuous change in the position of the sun.

QN is parallel to the polar axis when the porte lumière is in position. The angle between *LM* and *QN* changes with the sun's angular distance from the equator, called its declination. If the declination is represented by *d*, then the angle between *ML* and *QN* equals $\frac{90^\circ - d}{2}$. It will be found convenient to graduate the piece, *K*, to degrees to aid in setting *LM*.

144. Laboratory Seconds Clock. — In the absence of a clock with electrical connections to give time signals, the following cheap and simple device may be used: —

Procure a brass or iron rod about 1.02 m. long and 0.5 cm. in diameter, a large iron ball about 8 or 10 cm. in diameter with a hole through it of the diameter of the rod, and a piece of watch spring. Thread one end of the rod, and fit to it a small nut. Slot the other end, and rivet in it a piece of watch spring, having it extend about 5 cm. beyond the rod. Support the pendulum from an iron bracket screwed to the side wall. A slotted bar of brass soldered to the bracket with the end of the watch spring riveted in the slot will be a simple way to attach the pendulum. Solder to the lower end of the brass rod a piece of platinum wire about 1 mm. in diameter and flattened at the end. This piece need not be over 1.5 cm. long. Provide an electrical contact for the pendulum by a globule of mercury beneath it, through which the platinum strip passes at each vibration. An efficient mercury contact can be made out of a block of wood 2 × 5 × 5 cm. by boring a small hole through it parallel to the face of the block, closing one end with an iron binding post, and the other with an iron machine screw. A narrow slot is cut in the face of the block, connected at its center with the hole through the block, and filled with mercury. By turning the machine screw, a ridge of mercury can be made to stand up above the face of the block.

Place the pendulum in circuit with a battery and a telegraph sounder. Adjust it to seconds by turning the nut beneath the ball. When set in vibration, it will continue to swing for two or three hours.

APPENDIX B.

LABORATORY OPERATIONS.

145. Cutting Glass. — To cut small glass tubes, make a deep scratch with a three-cornered file across the tube as it rests on the table; then holding the tube, as shown in Fig. 115, with the scratch turned from you, press suddenly outward with the thumbs, and it will break as desired. To cut off large glass tubes, flasks, bottles, etc., make a deep scratch with a file, then apply to it either a pointed piece of glowing charcoal, a heated metal rod, or a fine gas jet. The sudden expansion by heat will generally produce a crack; if not, then touch the heated spot with a wet stick. A crack thus started may be led in any desired direction by keeping the source of heat moving slowly a few millimeters in advance of it. To get a small-pointed gas flame for the above purpose, connect a glass jet tube to the gas supply by a piece of rubber tubing.

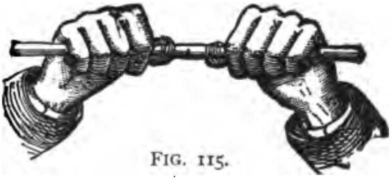

FIG. 115.

An excellent way to cut large glass tubes is to scratch them on the inside. Diamond points attached to stout metal rods can be purchased of apparatus dealers for this purpose. A cheap substitute is to be had in a hard steel wheel pivoted near the end of a rod. A glass tube, thus scratched on the inside, will almost invariably break off on heating it in a Bunsen flame.

Glass plates are readily cut with the common "glass cutter," an instrument consisting of a hard steel wheel set in the end of a metal rod. Glass disks are readily cut by an instrument consisting of a pivoted arm, on which is clamped a short rod carrying a steel wheel or diamond point.

146. Smoothing the End of a Glass Tube. — Hold the end of the tube in the edge of a Bunsen or spirit flame, slowly turning it around. Remove from the flame soon after the flame becomes a bright yellow. In the case of large tubes it will be necessary to smooth them by grinding on a smooth flagstone, wet with water.

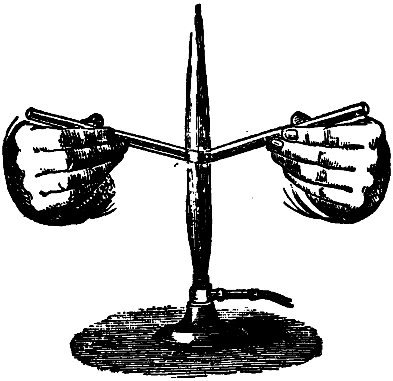

FIG. 116.

147. Bending Tubes. — Warm the tube for several inches each side of the place where the bend is to be by slowly passing it through the flame; then bring the tube into the gas flame, slowly rotating it on its axis, heating it evenly for 3 or 4 cm. When the flame becomes a bright yellow the tube will be soft. Then gently bend the ends from you until the required angle is obtained. Figure 116 shows how to hold the tube. In bending moderately large tubes, a flat flame is preferable. Figure 117 exhibits an attachment for the Bunsen burner to produce such a flame. The fish-tail gas burner gives an excellent flame for use in bending small tubes. To bend heavy tubes the blowpipe will be needed.

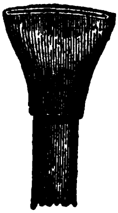

FIG. 117.

148. Closing the End of a Tube. — Soften the tube at the end by holding it in the flame, and then pull the end out with another piece of glass. Keep removing the tail that is left till it becomes quite small; then heat the end for a few minutes, turning the tube in the fingers. If the tube is held in an inclined position in the flame, the opening will keep contracting till finally it closes up.

149. Drawing out Tubes. — Thoroughly soften the tube uniformly for about 3 cm. of its length, then remove from the flame and pull the parts asunder till the diameter is about 1 mm., holding the tube steadily till it cools, to avoid crooking it. Now scratch it with a file at the smallest part and break it in two, smoothing the ends in the flame.

150. Drilling Holes in Glass. — Small holes are quite readily drilled through glass by using a hard drill wet with a solution of camphor in oil of turpentine. A three-cornered file, with the end broken off, makes a good drill for glass. Large holes may be drilled with a brass tube and emery powder moistened with water. A piece of wood, with a hole in it of the required size, cemented to the glass plate, makes an excellent guide for the drill. The tube can be rotated by the fingers, or if a wooden pulley is attached to it, the common drill-stock bow may be used. If the drill is moistened with a paste of fluor spar and sulphuric acid, the labor of drilling glass is much lessened.

151. Useful Cements. — The following cements will be found very useful, both in making and in repairing apparatus: —

For cementing wood, leather, metal, or glass to glass, melt, at 100° C., one part of gutta percha and one part of pine pitch, stirring till homogeneous. Soften the cement by heat when needed for use.

An excellent cement for use about electrical apparatus may be made by melting together the following: rosin, 7 oz., beeswax, 5 oz., Venetian red, 4 oz., and Venice turpentine, 1½ oz. Less Venice turpentine will make it harder.

152. Cleaning Mercury. — Fit a double perforated rubber stopper to a cylindrical glass jar about 1¼ in. in diameter and 8 in. deep. Through this stopper pass two glass tubes, one reaching nearly to the bottom of the jar and the other just through the stopper. Connect the latter tube to a Chapman aspirator. Between the stopper and the bottom of the jar insert an electric light carbon; one free from copper must be used. Now fill the cylinder about half full of mercury, and then pour in dilute sulphuric acid (one part acid to ten of water) till covered to the depth of about an inch. Introduce the carbon rod, insert the stopper, connect the aspirator to the water faucet, and draw air through the mercury and acid for at least half an hour. Electrolytic action is set up between the carbon and the metal impurities of the mercury, the salts produced going into solution. Pour off the acid solution, add fresh acid, and continue the action for a half hour longer. Mercury badly alloyed may require this operation to be repeated several times. After pouring off the acid wash the mercury through several changes of water, to remove all traces of acid, and then dry thoroughly.

153. Soldering. — Prepare a soldering fluid by adding scraps of zinc to hydrochloric acid till it refuses to dissolve any more. Wet the surfaces to be joined with this fluid, place between them a small piece of soft solder; an alloy of lead and tin; then apply heat either by holding the article in a flame, or by directing upon it a blowpipe flame; when cool the surfaces will be found firmly joined together.

154. To amalgamate Battery Zincs. — Immerse the zinc for a few minutes in the following fluid, then remove, and wash in water: Dissolve 15 $cm.^3$ of mercury in a mixture of 170 $cm.^3$ of nitric acid, and 625 $cm.^3$ of hydrochloric acid. The amalgamating fluid should be kept in a glass-stoppered bottle.

155. Battery Fluid for the Carbon and Zinc Battery. — No. 1. Dissolve 4 oz. of chromic acid in one quart of dilute sulphuric acid, 1 volume of sulphuric acid to 12 of water, and allow to cool and settle.

No. 2. Dissolve 7 oz. of sodium bichromate in one pint of water, to which is added 3¼ oz. of sulphuric acid.

156. Care of Batteries. — All the connections in a battery must be kept clean and bright, and the zincs well amalgamated. The porous cups must be thoroughly soaked in water before using, and kept in water while the battery is idle, if immediate action is wanted on putting the battery together. In the Daniell cell, when the porous cup becomes clogged with copper, it can be renovated by soaking in nitric acid. In the carbon battery, using the bichromate or chromic acid solution, the carbons should be soaked in hot paraffin. When cold, that clinging to the outside should be scraped off. The paraffin now fills the pores of the carbon, and prevents the chromic salts working up through the pores of the carbon and corroding the clamp at the top.

Carbon plates can be soldered to brass or copper by electroplating them with copper. To do this, connect the carbon plate to the negative pole of two or more Daniell cells joined in series, and a strip of copper to the positive pole. Let these electrodes dip close together, but not in contact, in a vessel containing a solution of copper sulphate, removing them when the plating covers completely the surface to be plated. Cover with paraffin the surface not to be plated.

157. Electrical Amalgam. — To insure the electrification of a glass rod by rubbing with silk, coat the silk with a very thin layer of lard, and then sprinkle over it powdered bisulphide of tin. The rubbers of a frictional machine may be treated in the same manner.

158. Operating Electrical Machines. — Remove all dust by wiping every part with a cotton cloth wet with kerosene oil. In the case of the old form of the Holtz machine, coat the plates with kerosene oil, and pour some on the base of the machine that its vapor may form an insulating atmosphere about the parts of the machine. Treated in this manner, the frictional machine and the old form of Holtz machine are made much more efficient, and are rendered almost independent of weather conditions.

The same treatment applied to the Voss and the Wimshurst greatly improves their action. Frequently self-charging machines, after standing idle for a long time, will not start up. In that case excite a glass (flint) tube by friction and charge by contact two or three of the buttons on the revolving plate. Then, on rotating the machine, it will start up at once.

APPENDIX C.

PHYSICAL CONSTANTS.

TABLE I.

Capillarity.

Height at 0° C. to which the liquid will rise in a glass tube 2 mm. in diameter. The height for any temperature between the limits is found by substituting the temperature for t in the third column.

Liquid	Density	Height in Mm.	Limits of Temperature
Alcohol, ethyl	0.8208	$6.050 - 0.0116\,t - 0.000051\,t^2$	0° to 75° C.
Ether . . .	0.7370	$5.400 - 0.0254\,t$	– 6° to 35° C.
Olive oil . .	0.9150	$7.461 - 0.0105\,t$	15° to 150° C.
Sulphuric acid	1.840	$8.400 - 0.0153\,t - 0.000094\,t^2$	12° to 90° C.
Turpentine .	0.8902	$6.760 - 0.0167\,t$	17° to 137° C.
Water . . .	1.0000	$15.332 - 0.0286\,t$	0° to 82° C.

TABLE II.

Densities of Various Substances.

The following table gives the mass in grams of 1 cm.^{3} of the substance: —

Acetic acid	1.060
Agate	2.5 to 2.7
Alcohol, ethyl	0.791
Alcohol, methyl	0.810
Alum, potassium . .	1.71 to 1.757
Aluminium	2.56 to 2.80
Amber	1.06 to 1.11
Antimony, cast . .	6.70 to 6.72
Apple-tree wood . .	0.66 to 0.84
Asbestos	2.0 to 2.8
Ash wood	0.65 to 0.85
Basswood	0.32 to 0.59
Beech wood . . .	0.70 to 0.90

Beeswax	0.964
Benzine	0.72 to 0.74
Benzene	0.899
Birch wood	0.51 to 0.77
Bismuth, cast	9.70 to 9.90
Blood, human	1.060
Borax	1.7 to 1.8
Boxwood	0.95 to 1.16
Brass	8.20 to 8.60
Brick	1.6 to 2.4
Bromine	3.187
Butter	0.86 to 0.94
Calcium chloride	2.230
Camphor	0.986 to 0.996
Carbon disulphide	1.293
Carbon dioxide, liquid	0.947
Cedar	0.49 to 0.57
Chalk	1.8 to 2.8
Cherry wood	0.70 to 0.90
Chestnut wood	0.535 to 0.606
Chloroform	1.48 to 1.525
Clay	1.8 to 2.6
Coal, hard	1.26 to 1.80
Coal, soft	1.27 to 1.42
Concrete	1.9 to 2.2
Copper	8.80 to 8.95
Cork	0.18 to 0.24
Corundum	3.9 to 4.0
Diamond	3.5 to 3.6
Earth, dry	1.52 to 2.00
Ebonite	1.15
Ebony	1.11 to 1.33
Elm wood	0.54 to 0.60
Emerald	2.770
Emery	3.900
Ether	0.736
Feldspar	2.53 to 2.60
Fir, spruce	0.48 to 0.70
Flint	2.63
Fluor spar	3.14 to 3.20
Galena	7.3 to 7.6
Garnet	3.6 to 3.8
German silver	8.30 to 8.45
Glass, crown	2.4 to 2.8
Glass, flint	2.9 to 4.5
Glycerine	1.260
Gold	19.26 to 19.34
Granite	2.5 to 3.0
Graphite	1.9 to 2.5
Gutta-percha	0.966
Gypsum, crys.	2.26 to 2.32
Heavy spar	4.430
Hickory wood	0.60 to 0.93
Honey	1.450
Human body, mean	1.07
Ice	0.88 to 0.92
Iceland spar	2.6 to 2.8
India rubber	0.930
Iodine	4.95
Iron	7.03 to 7.90
Iron pyrites	5.00
Ivory	1.83 to 1.92
Lard	0.947
Lead	11.36 to 11.40
Lignum vitæ	1.17 to 1.333
Lime, quick	0.843
Limestone	2.46 to 2.86
Magnesium	1.69 to 1.75
Magnetite	4.9 to 5.2
Mahogany	0.56 to 0.852
Maple wood	0.62 to 0.75
Marble	2.5 to 2.8
Mercury	13.596
Mica	2.6 to 3.2
Milk	1.032
Molasses	1.426
Mortar, average	1.700
Naphtha, wood	0.810 to 0.848
Naphtha, petroleum	0.665

Nickel	8.30 to 8.90
Nitric acid	1.38 to 1.559
Oak wood	0.60 to 0.90
Oil, castor	0.970
Oil, linseed	0.940
Oil, olive	0.915
Oil, turpentine	0.870
Oil, whale	0.923
Paraffin	0.824 to 0.940
Pear-tree wood	0.61 to 0.73
Petroleum	0.836
Phosphorus	1.830
Pine wood	0.461 to 0.590
Pitch	1.150
Platinum	21.20 to 21.70
Poplar wood	0.35 to 0.5
Porcelain	2.3 to 2.5
Potassium	0.86 to 0.88
Quartz	2.65
Resin	1.07
Rock salt	2.28 to 2.41

Sal ammoniac	1.5 to 1.6
Saltpetre	1.95 to 2.08
Sand	1.40 to 1.65
Sandstone	2.2 to 2.5
Silver	10.424 to 10.57
Slate	2.6 to 2.88
Sodium	0.95 to 0.97
Steel, unhammered	7.816
Sugar cane	1.593
Sulphur, native	2.033
Sulphuric acid	1.840
Tallow	0.91 to 0.97
Tar	1.015
Tin	7.29 to 7.30
Tourmaline	2.94 to 3.24
Walnut wood	0.64 to 0.70
Water, sea	1.027
Wax, white	0.970
White metal, Babbitt	7.310
Willow wood	0.40 to 0.60
Zinc	7.04 to 7.19

The following table gives the specific gravity of gases and vapors, air under the standard conditions being taken as the standard. The density is found by multiplying the specific gravity by 0.0012932.

Air	1.000
Ammonia	0.5967
Carbon monoxide	0.9670
Carbon dioxide	1.5241
Chlorine	2.4501
Coal gas	0.34 to 0.65
Cyanogen	1.8060
Hydrochloric acid	1.2780
Hydrogen	0.0693

Hydrogen disulphide	1.1912
Marsh gas	0.5540
Nitrogen	0.9714
Nitrogen binoxide	1.0392
Nitrogen monoxide	1.5241
Oxygen	1.1057
Steam at 100° C.	0.469
Sulphur dioxide	2.2113

TABLE III.

Density of Water and Mercury.

TEMP. C.	WATER	MERCURY	TEMP. C.	WATER	MERCURY
– 10°	0.998145	13.6203	26°	0.99687	13.5316
– 9	8427	6178	27	660	5292
– 8	8685	6153	28	633	5267
– 7	8911	6129	29	605	5243
– 6	9118	6104	30	0.99577	13.5218
– 5	0.999298	13.6079	31	547	5194
– 4	9455	6055	32	517	5169
– 3	9590	6030	33	485	5145
– 2	9703	6005	34	452	5120
– 1	9797	5981	35	0.99418	13.5096
0	0.999871	13.5956	36	383	5071
1	9928	5931	37	347	5047
2	9969	5907	38	310	5022
3	9991	5882	39	273	4998
4	1.000000	5857	40	0.99235	13.4974
5	0.999990	13.5833	41	197	
6	9970	5808	42	158	
7	9933	5783	43	118	
8	9886	5759	44	078	
9	9824	5734	45	0.99037	
10	0.999747	13.5709	46	8996	
11	9655	5685	47	954	
12	9549	5660	48	910	
13	9430	5635	49	865	
14	9299	5611	50	0.98820	13.4731
15	0.999160	13.5586	55	582	
16	9002	5562	60	338	13.4488
17	8841	5537	65	074	
18	8654	5513	70	7794	13.4246
19	8460	5488	75	0.97498	
20	0 998259	13.5463	80	194	13.4005
21	8047	5439	85	6879	
22	7826	5414	90	556	13.3764
23	7601	5390	95	219	
24	7367	5365	100	0.95865	13.3524
25	0.997120	13 5341			

TABLE IV.

Electrical Resistance.

This table gives the resistance in ohms at 0° C.

	1 m. long and 1 mm.² cross-section	100 ft. long and $\frac{1}{1000}$ in. in diameter
Aluminium, annealed	2.906×10^{-2}	17.48×10^{2}
Brass	5.72×10^{-2}	
Copper, annealed	1.584×10^{-2}	9.529×10^{2}
German silver	20.89×10^{-2}	125.7×10^{2}
Gold, annealed	2.088×10^{-2}	12.56×10^{2}
Iron, annealed	9.693×10^{-2}	58.31×10^{2}
Lead, pressed	19.14×10^{-2}	115.1×10^{2}
Mercury	94.07×10^{-2}	565.9×10^{2}
Nickel, annealed	12.43×10^{-2}	74.78×10^{2}
Phosphor bronze	9.32×10^{-2}	382.156×10^{2}
Platinum, annealed	9.035×10^{-2}	54.35×10^{2}
Platinoid	47.92×10^{-2}	288.25×10^{2}
Silver, annealed	1.460×10^{-2}	8.781×10^{2}
Tin, pressed	13.18×10^{-2}	79.29×10^{2}
Zinc, pressed	5.613×10^{-2}	33.76×10^{2}

TABLE V.

Electromotive Force of Primary Batteries.

Bunsen, amal. zinc, H_2SO_4 + 12 Aq, HNO_3 of density 1.38	1.86 volts.
Carhart-Clark standard cell	1.44 volts.
Chloride of silver	1.1 volts.
Chromic acid	2.0 volts.
Daniell, amal. zinc, H_2SO_4 + 12 Aq, sat. sol. of $CuSO_4$ + 5 H_2O	1.09 volts.
Daniell, amal. zinc, equi-dense solutions of $ZnSO_4$ and $CuSO_4$	1.1 volts.
Diamond carbon cell	1.36 volts.

Dry cell (Gassner)	1.3 volts.
Edison-Lalande	0.7 volt.
Leclanché	1.5 volts.
Taylor's	1.9 volts.
Voltaic couple, simple	0.98 volt.

TABLE VI.

Electrical Carrying Capacity of Wire, B. and S. Gauge, in Amperes.

GAUGE NO.	COPPER EXPOSED TO AIR	GER. SILVER MAXIMUM SAFE	COPPER COVERED UP	TINNED IRON WIRE
0000	312	——	175	——
000	262	——	145	——
00	220	——	120	——
0	185	——	100	——
1	156	——	95	——
2	131	——	70	——
3	110	——	60	——
4	92.3	——	50	——
5	77.6	——	45	——
6	65.2	——	35	——
7	54.8	——	30	——
8	46.1	——	24	17.4
9	38.7	——	22	14.6
10	32.5	8.5	20	12.3
11	27.3	5.4	15	10.3
12	23.0	4.6	13	8.7
13	19.3	3.8	11	7.3
14	16.2	3.2	9	6.1
15	13.6	2.7	7	5.1
16	11.5	2.3	6	4.3
17	9.6	1.9	5	3.6
18	8.1	1.65	4.5	3.0

TABLE VI. — *Continued.*

GAUGE NO.	COPPER EXPOSED TO AIR	GER. SILVER MAXIMUM SAFE	COPPER COVERED UP	TINNED IRON WIRE
19	6.7	1.21	3.7	2.52
20	5.7	0.99	3.2	2.17
21	4.8	0.88	2.7	1.82
22	4.0	0.66	2.2	1.53
23	3.4	0.55	1.9	1.28
24	2.8	0.488	1.5	1.08
25	2.4	0.434	1.3	——
26	2.0	0.385	1.1	——
27	1.7	0.343	0.9	——
28	1.4	——	0.7	——
29	1.2	——	0.6	——
30	1.0	——	0.5	——
31	0.84	——	0.47	——
32	0.70	——	0.39	——
33	0.60	——	0.34	——
34	0.50	——	0.28	——
35	0.42	——	0.23	——
36	0.35	——	0.19	——
37	0.27	——	0.15	——
38	0.25	——	0.14	——
39	0.21	——	0.12	——
40	0.17	——	0.09	——

TABLE VII.

Electrical Resistance, Diameter, Area, Etc., of Copper Wire, B. and S. Gauge, Temperature, 24° C.

Gauge No	Size					Weight		Resistance				
	Diam. In.	Diam. Mm.	Area Sq. In.	Area C.M.	Area Cm.²	Lbs. per 1000 feet	Feet per Lb.	Ohms per 1000 feet	Feet per Ohm	Ohms per Lb.	Ohms per 1000 m.	Meters per Ohm
0000	.4600	11.684	.16619	211600	.8107	639.60	1.564	0.0463	21601	.0000724	0.1519	6584
000	.4096	10.405	.13179	167805	.7012	507.22	1.971	0.0584	17131	.000115	0.1915	5221
00	.3648	9.266	.10459	133079	.6136	402.25	2.486	0.0736	13586	.000183	0.2415	4141
0	.3249	8.251	.08293	105592	.5319	319.17	3.133	0.0928	10774	.000291	0.3045	3284
1	.2893	7.348	.06573	83694	.4560	252.98	3.952	0.1170	8544	.000462	0.3840	2604
2	.2576	6.543	.05213	66373	.3858	200.63	4.994	0.1476	6775	.000737	0.4842	2065
3	.2294	5.827	.04134	52633	.3218	159.09	6.285	0.1861	5373	.001169	0.6106	1638
4	.2043	5.189	.03278	41743	.2727	126.17	7.925	0.2347	4261	.001859	0.7699	1299
5	.1819	4.621	.02600	33102	.2277	100.05	9.995	0.2959	3379	.002959	0.9709	1030
6	.1620	4.115	.02062	26250	.1868	79.34	12.604	0.3731	2680	.004703	1.224	816.9
7	.1443	3.665	.01635	20816.7	.1570	69.92	15.893	0.4705	2125	.00748	1.544	647.8
8	.1285	3.264	.01277	16509.7	.1297	49.90	20.040	0.5933	1685	.01189	1.947	513.7
9	.1144	2.906	.01028	13094.2	.1051	39.58	25.265	0.7482	1337	.01889	2.455	407.4
10	.1014	2.588	.00815	10381.6	.0830	31.38	31.867	0.9434	1060	.03006	3.095	323.1
11	.0907	2.305	.00647	8234.1	.0682	24.89	40.176	1.190	840.6	.04792	3.903	256.2
12	.0808	2.053	.00513	6529.9	.0548	19.74	50.659	1.500	666.6	.07599	4.922	203.2
13	.0720	1.828	.00407	5178.4	.0429	15.65	63.898	1.892	528.7	.12086	6.206	161.1

14	.0641	1.628	.00315	4106.8	.0324	12.41	80.580	2.385	419.2	.19222	7.826	127.8
15	.0571	1.450	.00256	3256.8	.0263	9.84	101.626	3.008	332.5	.30564	9.868	101.3
16	.0508	1.2908	.00203	2582.7	.0208	7.81	128.041	3.793	263.7	.48555	12.44	80.37
17	.0453	1.1495	.00161	2048.2	.0159	6.19	161.551	4.783	209.1	.77260	15.69	63.73
18	.0403	1.0237	.00128	1624.3	.0117	4.91	203.666	6.031	165.8	1.222	19.79	50.54
19	.0354	0.9116	.00098	1288.1	.0081	3.786	264.136	7.604	131.5	2.009	24.95	40.08
20	.0320	0.8168	.00080	1021.4	.00657	3.086	324.045	9.589	104.3	3.107	31.46	31.79
21	.0285	0.7249	.00064	810.09	.00519	2.448	408.497	12.09	82.70	4.939	39.67	25.21
22	.0253	0.6438	.00050	642.47	.00397	1.942	514.933	15.25	65.59	7.851	50.02	19.99
23	.0226	0.5733	.00040	509.45	.00292	1.539	649.773	19.23	52.01	12.493	63.08	15.85
24	.0201	0.5106	.00032	404.01	.00245	1.221	819.001	24.24	41.25	19.854	79.54	12.57
25	.0179	0.4545	.00025	320.41	.00203	0.967	1034.126	30.57	32.71	31.615	100.30	9.97
26	.0159	0.4049	.000199	254.08	.00164	0.768	1302.083	38.55	25.94	50.196	126.47	7.907
27	.0142	0.3606	.000158	201.50	.00136	0.608	1644.737	48.61	20.57	79.958	159.48	6.270
28	.0126	0.3211	.000125	159.80	.00111	0.484	2066.116	61.30	16.31	126.68	201.10	4.973
29	.0113	0.2859	.000100	126.72	.00094	0.384	2604.167	77.29	12.94	201.25	253.58	3.943
30	.0100	0.2546	.000079	100.50	.000779	0.302	3311.258	97.46	10.26	322.73	319.76	3.127
31	.0089	0.2268	.000063	79.71	.000682	0.239	4184.100	122.9	8.137	514.21	403.2	2.480
32	.0079	0.2019	.000050	63.20	.000591	0.190	5263.158	155.0	6.452	815.74	508.4	1.967
33	.0071	0.1798	.000039	50.13	.000507	0.151	6622.517	195.4	5.117	1294.2	641.1	1.560
34	.0063	0.1601	.000031	39.74	.000429	0.121	8264.463	246.4	4.058	2036.6	808.5	1.237
35	.0056	0.1426	.000025	31.52	.000357	0.094	10638.30	310.7	3.218	3305.9	1019.4	0.981
36	.0050	0.1270	.000020	25.00	.000293	0.075	13333.33	391.8	2.552	5263.8	1285.5	0.778
37	.0045	0.1131	.000016	19.83	.000234	0.060	16666.66	494.1	2.024	8234.5	1621.0	0.617
38	.0040	0.1007	.000012	15.72	.000182	0.045	22222.22	623.0	1.605	13845.6	2044.0	0.489
39	.0035	0.0897	.000010	12.47	.000137	0.038	26315.79	785.6	1.273	20672.3	2577.5	0.388
40	.0031	0.0799	.000008	9.88	.000117	0.030	33333.33	990.6	1.009	33038.9	3250.1	0.308

TABLE VIII.

Acceleration due to Gravity.

	CM. PER SEC. PER SEC.
Ann Arbor, Mich., lat. 42° 16′	980.29
Berlin, Germany, lat. 52° 30′	981.26
Boston, Mass., lat. 42° 41′	980.38
Chicago, Ill., lat. 41° 47′	980.26
Denver, Col., lat. 39° 40′	979.59
Equator, at sea level	977.99
Kew, England, lat. 51° 28′	981.20
Latitude of 45°, at sea level	980.60
Montreal, Can., lat. 45° 31′	980.73
New York, lat. 40° 43′	980.19
Paris, lat. 48° 50′	980.96
Pole, at sea level	983.21
San Francisco, lat. 37° 47′	979.95
Washington, lat. 38° 53′	980.10

TABLE IX.

Boiling Points of Liquids at Barometric Press. 76 Cm.

	°C.		°C.
Acetic acid	119	Carbon dioxide	−78.0
Acetone	56.3	Carbon disulphide	48.0
Alcohol (amyl)	131.8	Ether	34.9
Alcohol (ethyl)	78.4	Mercury	357.0
Alcohol (methyl)	66.5	Nitrous oxide	−92.0
Aldehyde	20.8	Petroleum	40 to 70.0
Ammonia (liquid)	−39.0	Sulphuric acid	338.0
Aniline	183.7	Sulphur dioxide	−8.0
Benzine	90 to 110.0	Turpentine	159.3
Benzole	80.4	Water, distilled	100.0
Bromine	63.0	Water, saturated with NaCl	102.0

TABLE X.

Mean Coefficients of Expansion between 0° and 100° C.

LINEAR.

Aluminium	0.00002221	Lead	0.00002799
Antimony	0.00000980	Marble	0.00000786
Bismuth	0.00001330	Paraffin (0°–49°)	0.00020041
Brass, wire	0.00001856	Pine	0.00000496
Bronze	0.00001844	Platinum	0.00000886
Copper	0.00001866	Sandstone, red	0.00001174
Ebonite	0.00008420	Silver	0.00001943
Glass, tube	0.00000833	Sulphur	0.00006413
Glass, Jena	0.00000810	Steel, tempered	0.00001322
Gold	0.00001460	Steel, untempered	0.00001095
Graphite	0.00000786	Tin	0.00002296
Iron, wire drawn	0.00001235	Zinc	0.00002976
Iron, wrought	0.00001220		

CUBICAL.

Acetic acid, 0° to 100°	0.001172	Ether, 0° to 20°	0.001561
Acetone, 0° to 100°	0.00135	Glycerine, 0° to 100°	0.00053
Air, const. vol., 0° to 100°	0.003663	Ice, –20° to –1°	0.0001125
Air, const. pres. 0° to 100°	0.003667	Mercury, 0° to 100°	0.000181
Alcohol, amyl, 0° to 100°	0.001077	Olive oil, 0° to 100°	0.000803
Alcohol, ethyl, 0° to 50°	0.001087	Sulphuric acid, 0° to 100°	0.005785
Alcohol, methyl, 0° to 60°	0.001248	Turpentine, 0° to 100°	0.001051
Benzene, 11° to 81°	0.001385	Water, 4° to 100°	0.000449
Carbon disulp., 0° to 70°	0.001252		

TABLE XI.

Heats of Fusion and Vaporization.

FUSION.

Beeswax	42.3	Lead	5.86
Bismuth	12.64	Mercury	2.83
Iron, gray cast	23.	Phosphorus	4.97

Platinum	27.18	Sulphur	9.37
Potassium nitrate	48.9	Tin	14.25
Silver	21.07	Water	79.24
Sodium nitrate	64.87	Zinc	28.13
Spermaceti	36.98		

VAPORIZATION.

Acetic acid, glacial	102	Ether	90.4
Alcohol, ethyl	207	Mercury	62.0
Alcohol, methyl	264	Sulphur	362.0
Bromine	45.6	Turpentine	74.0
Carbon disulphide	86.7	Water	535.9

TABLE XII.

Melting Points.

Aluminium	600° to 850° C.	Mercury	−38°.5 to −39°.44 C.
Antimony	425° to 450°	Nickel	1450 to 1600
Beeswax	52	Paraffin	38 to 52
Bismuth	270	Phosphorus	44.2
Brass	1020	Platinum	1775 to 2200
Butter	33	Potassium	55 to 63.0
Butter, cacao	33.5	Rose's metal	93.75
Butter, cocoa	20.0	Silver	916 to 1040
Copper	1050 to 1330	Sulphur	115
German silver	1093	Sodium	97.6
Glass, crown	400	Spermaceti	43.5 to 44.1
Gold	1035 to 1250	Stearic acid	69.9
Iridium	1950 to 2500	Stearin	60
Iron, pure	1500 to 1800	Tallow, fresh	43
Japan wax	50.4 to 51	Tin	226.5 to 235
Lard	27 to 33.2	Wax, white	65
Lead	322 to 335	Wood's metal	60.5
Margaric acid	59.9	Zinc	400 to 433

TABLE XIII.

Specific Heats.

Acetic acid	0.659	Iodine, 9° to 98°	0.054
Acetone, 20° to 41°	0.530	Iron, 15° to 100°	0.1123
Air, 0° to 100°	0.237	Iron, cast, 0° to 100°	0.1286
Alcohol, ethyl, 0° to 50°	0.615	Lead, 10° to 100°	0.0314
Alcohol, methyl, 0° to 61°	0.613	Marble	0.2129
Aluminium, 15° to 97°	0.212	Mercury, 0° to 100°	0.0330
Antimony, 0° to 100°	0.0507	Nickel, 14° to 97°	0.1092
Beeswax, 0° to 58°	0.665	Nitrogen, const. pres.	0.2438
Benzene, 11° to 81°	0.450	Olive oil	0.504
Bismuth, 9° to 100°	0.0298	Oxygen, const. pres.	0.2175
Brass, hard, 15° to 98°	0.0858	Platinum, 0° to 100°	0.0323
Carbon disulphide, 34° to 60°	0.221	Salt	0.173
Copper, 15° to 100°	0.0933	Silver, 0° to 100°	0.0557
Ether, 0° to 33°	0.517	Steel	0.118
German silver	0.0944	Sulphur, 15° to 97°	0.1844
Glass, thermometer, 0° to 100°	0.177	Tin, 0° to 100°	0.0559
Glycerine, 0° to 100°	0.555	Turpentine, 0° to 100°	0.426
Hydrogen, const. pres.	3.409	Water, 0° to 100°	1.0050
Ice, – 30° to 0°	0.505	Zinc, 0° to 100°	0.0935

TABLE XIV.

Indices of Refraction.

The following indices are given for the mean D line:—

Air	1.000294	Glycerine	1.47
Aqueous humor, eye	1.357	Ice	1.310
Alcohol, ethyl	1.361	Iceland spar, ordinary ray	1.654
Alcohol, methyl	1.333	Iceland spar, extra ray	1.483
Benzene	1.5029	Quartz, ordinary ray	1.544
Canada balsam	1.54	Quartz, extra ray	1.553
Carbon disulphide	1.643	Rock salt	1.550
Crown glass	1.516 to 1.5604	Ruby	1.779
Crystalline lens, eye	1.384	Turpentine	1.474
Diamond	2.47 to 2.75	Vitreous humor, eye	1.339
Ether	1.354	Water	1.332
Flint glass	1.541 to 1.792		

TABLE XV.

Mensuration Rules.

Area of triangle = base × ½ altitude.
Area of parallelogram = base × altitude.
Area of trapezoid = altitude × ½ sum of parallel sides.
Circumference of circle = diameter × 3.1416.
Diameter of circle = circumference × 0.3183 = $\sqrt{\text{area} \div 0.7854}$.
Area of circle = diameter squared × 0.7854.
Area of ellipse = product of diameters × 0.7854.
Area of regular polygon = sum of sides × ½ apothem.
Lateral surface of cylinder = circumference of base × altitude.
Volume of cylinder = area of base × altitude.
Surface of sphere = diameter × circumference.
Volume of sphere = diameter cubed × 0.5236.
Surface of pyramid
Surface of cone } = circumference of base × ½ slant height.
Volume of cone = area base × ⅓ altitude.
Surface of frustum of pyramid or cone = sum of circumference of bases × ½ slant height.
Volume of frustum of pyramid or cone = ⅓ altitude × sum of area of bases and square root of product of these areas.

TABLE XVI.

Lengths of Seconds Pendulum.

Greenwich, latitude 51° 29′	99.416 cm.
Paris, lat. 48° 50′	99.391 cm.
New York, lat. 40° 43′	99.314 cm.
Washington, lat. 38° 53′	99.304 cm.
Ann Arbor, Mich., lat. 42° 16′	99.323 cm.
Chicago, Ill., lat. 41° 47′	99.315 cm.
Denver, Col., lat. 39° 40′	99.253 cm.
San Francisco, lat. 37° 47′	99.289 cm.
Latitude, sea level, 45° 00′	99.356 cm.
Equator, sea level	99.091 cm.
Pole, sea level	99.620 cm.

TABLE XVII.

Velocity of Sound at 0° C.

	Meters		Meters
Air, per sec.	332	Hydrogen, per sec.	1269
Aluminium, per sec.	5104	Iron, per sec.	5127
Ash, with fiber, per sec.	4668	Iron at 20°, per sec.	5130
Beech, with fiber, per sec.	3340	Lead, per sec.	1228
Brass, per sec.	3500	Maple, per sec.	4106
Caoutchouc, per sec.	60	Oak, per sec.	3847
Carbon monoxide, per sec.	337	Oxygen, per sec.	317
Carbon dioxide, per sec.	261	Pine, per sec.	3322
Cedar, per sec.	5030	Silver, per sec.	2607
Chlorine, per sec.	206	Steel, cast, at 20°, per sec.	4990
Copper at 20°, per sec.	3560	Tallow, per sec.	357
Elm, per sec.	4120	Tin, per sec.	2500
Ether, per sec.	1159	Turpentine at 24°, per sec.	1212
Fir, per sec.	4638	Walnut, per sec.	4601
Gas, illuminating, per sec.	490	Water at 8°.1, per sec.	1435
Glass, per sec.	5026	Wax, per sec.	857
Gold at 20°, per sec.	1743	Zinc, per sec.	3700

TABLE XVIII.

Tenacity.

The following table gives the force in kilograms required to break a wire of the substance 1 mm. in diameter: —

Copper, drawn	40.3	Silver, drawn	29.00
Copper, annealed	30.54	Silver, annealed	16.02
Iron, drawn	61.10	Steel, drawn	70.00
Iron, annealed	46.88	Steel, annealed	40.00
Lead, drawn	2.07	Steel, cast, drawn	80.00
Lead, annealed	1.80	Steel, cast, annealed	65.75
Platinum, drawn	34.10	Tin, drawn	2.45
Platinum, annealed	23.5	Tin, annealed	1.70

TABLE XIX.

Trigonometrical Functions.

RULE.—To find the sine, tangent, etc., of an angle, find the degrees (integral part) in the column "angle," then proceed horizontally across the page to the column "sine," "tangent," etc., and write down the number found. To this add the product obtained by multiplying the difference between that number and the next one in order by the decimal part of the angle. When the angle exceeds 45°, the column giving the required function is marked at the bottom.

ANGLE	SINE		TANGENT		COTANGENT		COSINE		ANGLE
0	0.000	17	0.000	17			1.000	0	**90**
1	0.017	18	0.017	18	57.29		1.000	1	**89**
2	0.035	17	0.035	17	28.64		0.999	0	**88**
3	0.052	18	0.052	18	19.08		0.999	1	**87**
4	0.070	17	0.070	17	14.30		0.998	2	**86**
5	0.087	18	0.087	18	11.43		0.996	1	**85**
6	0.105	17	0.105	18	9.514		0.995	2	**84**
7	0.122	17	0.123	18	8.144		0.993	3	**83**
8	0.139	17	0.141	17	7.115	801	0.990	2	**82**
9	0.156	18	0.158	18	6.314	643	0.988	3	**81**
10	0.174	17	0.176	18	5.671	526	0.985	3	**80**
11	0.191	17	0.194	19	5.145	440	0.982	4	**79**
12	0.208	17	0.213	18	4.705	374	0.978	4	**78**
13	0.225	17	0.231	18	4.331	320	0.974	4	**77**
14	0.242	17	0.249	19	4.011	279	0.970	4	**76**
15	0.259	17	0.268	19	3.732	245	0.966	5	**75**
16	0.276	16	0.287	19	3.487	216	0.961	5	**74**
17	0.292	17	0.306	19	3.271	193	0.956	5	**73**
18	0.309		0.325		3.078		0.951		**72**
ANGLE	COSINE		COTANGENT		TANGENT		SINE		ANGLE

ANGLE	SINE		TANGENT		COTANGENT		COSINE		ANGLE
19	0.326	17	0.344	19	2.904	174	0.946	5	**71**
20	0.342	16	0.364	20	2.747	157	0.940	6	**70**
21	0.358	16	0.384	20	2.605	142	0.934	6	**69**
22	0.375	17	0.404	20	2.475	130	0.927	7	**68**
23	0.391	16	0.424	20	2.356	119	0.921	6	**67**
24	0.407	16	0.445	21	2.246	110	0.914	7	**66**
25	0.423	16	0.466	21	2.145	101	0.906	8	**65**
26	0.438	15	0.488	22	2.050	95	0.899	7	**64**
27	0.454	16	0.510	22	1.963	87	0.891	8	**63**
28	0.469	15	0.532	22	1.881	82	0.883	8	**62**
29	0.485	16	0.554	22	1.804	77	0.875	8	**61**
30	0.500	15	0.577	23	1.732	72	0.866	9	**60**
31	0.515	15	0.601	24	1.664	68	0.857	9	**59**
32	0.530	15	0.625	24	1.600	64	0.848	9	**58**
33	0.545	15	0.649	24	1.540	60	0.839	9	**57**
34	0.559	14	0.675	26	1.483	57	0.829	10	**56**
35	0.574	15	0.700	25	1.428	55	0.819	10	**55**
36	0.588	14	0.727	27	1.376	52	0.809	10	**54**
37	0.602	14	0.754	27	1.327	49	0.799	10	**53**
38	0.616	14	0.781	27	1.280	47	0.788	11	**52**
39	0.629	13	0.810	29	1.235	45	0.777	11	**51**
40	0.643	14	0.839	29	1.192	43	0.766	11	**50**
41	0.656	13	0.869	30	1.150	42	0.755	11	**49**
42	0.669	13	0.900	31	1.111	39	0.743	12	**48**
43	0.682	13	0.933	33	1.072	39	0.731	12	**47**
44	0.695	13	0.966	33	1.036	36	0.719	12	**46**
45	0.707	12	1.000	34	1.000	36	0.707	12	**45**
ANGLE	COSINE		COTANGENT		TANGENT		SINE		ANGLE

TABLE XX.

Logarithms of Numbers.

N.	0	1	2	3	4	5	6	7	8	9	D.
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	42
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	38
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	35
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	32
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	30
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	28
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	27
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	25
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	23
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	22
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	21
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	20
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	19
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	18
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	18
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	17
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	16
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	16
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	15
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	15
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	14
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	14
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	13
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	13
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	13
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	12
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	12
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	12
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	11
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	11
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	10
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	10
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	10
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	9
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	9
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	9
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	8
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	8
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	8

N.	0	1	2	3	4	5	6	7	8	9	D.
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	8
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	8
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	8
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	7
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	7
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	7
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	7
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	7
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	7
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	7
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	6
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	6
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	6
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	6
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	6
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	6
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	6
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	6
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	6
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	5
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	5
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	5
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	5
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	5
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	5
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	5
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	5
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	5
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	4

TABLE XXI.

Some Useful Numbers.

$\pi = 3.14159$ $\pi^2 = 9.86965$ $\frac{1}{\pi} = 0.31831$ $\sqrt{\pi} = 1.77245$	$\sqrt{2} = 1.4142$ $\sqrt{3} = 1.7320$ $\sqrt{5} = 2.2361$	$\sqrt{10} = 3.16228$ $\frac{1}{\sqrt{2}} = 0.7071$

Dyne at New York = 0 00102 gm.
Poundal at New York = 13825 dynes.
Erg = 0.00102 gm.-cm.
Ft.-lb. = 0.13825 kgm.-m.
Ft.-poundal = 421390 ergs.
Kgm.-m. = 7.23308 ft.-lb.
A cubic foot of water at 4° C. weighs 62.425 lb.
A cubic foot of water at 16⅔° C. weighs 62.321 lb.
A cubic foot of air at 0° C. weighs 0.080728 lb.
1 cm.^{8} of air at 0° C., 76 cm. bar. pres., weighs 0.0012932 gm.
1 l. of hydrogen at 0° C., 76 cm. bar. pres., weighs 0.08969 gm.
H. P. = 33000 ft.-lb. per minute.
H. P. = 7.452×10^9 ergs per minute.
1 Paris ft = 0.32484 m.
1 Paris line = 2.2588 mm.
1 German mile = 7.4204 km.
1 Rhenish ft. = 0.31385 m.

TABLE XXII.

Conversion Tables.

1. LENGTH.

TO REDUCE	MULTIPLY BY	TO REDUCE	MULTIPLY BY
Mi. to km. . . .	1.60935	Km. to mi.	0.62137
Mi. to m.	1609.347	M. to mi.	0.0006214
Yd. to m.	0.91440	M. to yd.	1.09361
Ft. to m.	0.30480	M. to ft.	3.28083
In. to cm. . . .	2.54000	Cm. to in.	0.39370
In. to mm. . . .	25.40005	Mm. to in.	0.03937

2. SURFACE.

TO REDUCE	MULTIPLY BY	TO REDUCE	MULTIPLY BY
Sq. yd. to m.^{2}	0.83613	Sq. m. to sq. yd.	1.19599
Sq. ft. to m.^{2}	0.09290	Sq. m. to sq. ft.	10.76387
Sq. in. to cm.^{2}	6.45163	Sq. cm. to sq. in.	0.15500
Sq. in. to mm.^{2}	645.163	Sq. mm. to sq. in.	0.00155

3. VOLUME.

TO REDUCE	MULTIPLY BY	TO REDUCE	MULTIPLY BY
Cu. yd. to m.^{3}	0.76456	Cu. m. to cu. yd.	1.30802
Cu. ft. to m.^{3}	0.02832	Cu. m. to cu. ft.	35.31661
Cu. in. to cm.^{3}	16.38716	Cu. cm. to cu. in.	0.06102
Cu. ft. to l.	28.31701	L. to cu. ft.	0.03532
Cu. in. to l.	0.01639	L. to cu. in.	61.02337
Gal. to l.	3.78543	L. to gal.	0.26417
Lb. of water to l.	0.45359	L. of water to lb.	2.20462

4. WEIGHT.

TO REDUCE	MULTIPLY BY	TO REDUCE	MULTIPLY BY
T. to kgm.	907.18486	Kgm. to T.	0.001102
Lb. to kgm.	0.45359	Kgm. to lb.	2.20462
Oz. to gm.	28.34953	Gm. to oz.	0.03527
Gr. to gm.	0.064799	Gm. to gr.	15.43236

5. FORCE, WORK, ACTIVITY, PRESSURE.

TO REDUCE	MULTIPLY BY	TO REDUCE	MULTIPLY BY
Lb.-wt. to dynes	444520.58	Dynes to lb.-wt.	22496×10^{-10}
Ft.-lb. to kgm.-m.	0.138255	Kgm.-m. to ft.-lb.	7.233
Ft.-lb. to ergs	13549×10^{3}	Ergs to ft.-lb.	0.7381×10^{-7}
Ft.-lb. to joules	1.3549	Joules to ft.-lb.	0.7381
Ft.-lb. per sec. to H.P.	18182×10^{-7}	H. P. to ft.-lb. per sec.	550
H. P. to watts	745.196	Watts to H. P.	0.001342
Lb. per sq. ft. to kgm. per m.^{2}	4.8824	Kgm. per m.^{2} to lb. per sq. ft.	0.2048
Lb. per sq. in. to gm. per cm.^{2}	70.3068	Gm. per cm.^{2} to lb. per sq. in.	0.01422
Poundals to lb.-wt.	0.031104	Lb.-wt. to poundals	32.15
Poundals to dynes	13826.37	Dynes to poundals	0.00007232

Calculated for $g = 980$ cm., or 32.15 ft. per sec. per sec.

6. MISCELLANEOUS.

To Reduce	Multiply by	To Reduce	Multiply by
Lb. of water to U.S. gal.	0.11983	U. S. gal. to lb. of water	8.345
Cu. ft. to U.S. gal.	7.48052	U.S. gal. to cu. ft.	0.13368
Lb. of water to cu. ft. at 4° C.	0.01602	Cu. ft. of water at 4° C. to lb.	62.425
Cu. in. to U.S. gal.	0.004329	U.S. gal. to cu. in.	231
Atmospheres to lb. per sq. in.	14.69640	Lb. per sq. in. to atmospheres	0.06737
Atmospheres to gm. per cm.^{2}	1033.296	Gm. per cm.^{2} to atmospheres	0.000968
Lb.-deg. F. to calories	252	Calories to lb.-deg. F.	0.003968
Calories to joules	4.18936	Joules to calories	0.2387
Mi. per hr. to ft. per sec.	1.46667	Ft. per sec. to mi. per hr.	0.68182
Mi. per hr. to cm. per sec.	44.704	Cm. per sec. to mi. per hr.	0.02237

APPENDIX D.

PRICE-LIST OF APPARATUS AND MATERIAL.

Simple Measurements.

Balance, beam, load 100 gm., sensitive to 1 mgm., without case	\$9.00 to \$15.00
Balance, beam, load 100 gm., sensitive to 1 mgm., with case	20.00
Balance, beam, load 1 kgm., sensitive to 1 cgm., without case	15.00
Balance, Jolly's	\$3.25 to 17.00
Caliper, inside and outside, 3 in.	.25
Caliper micrometer, metric	3.50
Caliper vernier, metric	2.50
Diagonal scale, metric, boxwood	.15
Dividers, 5 in.	\$0.25 to .40
Erdmann's float	.35
Gauge, tube	1.25
Gauge, wire, American standard	2.50
Graduate, 500 cm.^{8}, conical, Eng. and metric	1.30
Graduate, 250 cm.^{8}, conical, Eng. and metric	.90
Graduate, 100 cm.^{8}, cylindrical	.70
Magnifier	.30
Mercury, per lb	.80
Meter rod, metric side divided to mm., Eng. side to 0.1 in.	.25
Protractor, horn, 4 in. diam.	.12
Square-cornered blocks of wood, Ex. 21	.20
Weights, 1 mg. to 20 gm., in block	1.10
Weights, 1 mg. to 20 gm., in box, accurate	3.75
Weights, 1 cgm. to 1 kgm., in block	3.25

Elasticity.

Draw scale, 15 kgm. capacity	\$1.25
Elasticity of bending apparatus	2.00
Elasticity of torsion	6.00
Hooke's Law apparatus	1.50

Scale pan, brass, with cords	$.25
Weights, metric, iron, 10 gm. to 5 kgm.	3.00
Wire, brass, No. 27, per lb., Ex. 35	.60
Wire, brass, No. 30	.80

Mechanics of Solids.

Atwood's machine, Hawkes' patent	$40.00 to $50.00
Ball, iron, 2 kgm., Ex. 41	.40
Clamp, iron, 4 in.	.40
Clock, with seconds dial, electric contact, actuated by gravity	15.00
Clock, without dial	7.00
Draw scale, capacity 250 gm.	.55
Draw scale, capacity 2000 gm.	.45
Friction apparatus	.80
Graduated wooden bar, Ex. 40	.15
Inclined plane apparatus	4.00
Lever apparatus	1.50
Pendulum frame, simple	1.50
Pendulum frame, adjustable, with mercury contact	6.00
Protractor, paper, 0° – 360°, 4 in. diameter	.15
Pulleys, brass	.25
Telegraph sounder	.75
Wheel and axle	1.00

Mechanics of Fluids.

Air thermometer bulb	$ .25
Barometer, mercurial	5.75
Beaker, glass, 2 oz.	.10
Boyle's Law apparatus with mercury	$4.25 to 8.00
Capillary tubes, set of five, 10 cm. long	.10
Density bottle, adjusted, 25 $cm.^3$	.85
Density bottle, unadjusted	.10
Funnel, 4 in.	.15
Glass vessel, cylindrical, 60 cm. deep, Ex. 52	1.00
Hare's apparatus, Ex. 58	2.00
Iron ring-stand, three rings	.65
Metal cylinder for buoyancy	.25
Metal tubes, aluminium, Ex. 52	2.00

Pinchcock, Hoffman	$.30
Scale, V-shape at one end	.50
Siphon, glass	.25
Thermometer, Cent., paper scale	.75
Weights, 5 gm. each	.20
Universal wooden stand	1.25

Sound.

Battery for motor, per cell	$ 2.00
Chronograph	15.00
Cylindrical tubes, zinc, set of three, Art. 59.	1.50
Electric motor with speed counter attached	$5.00 to 7.00
Induction coil, 2.5 cm. spark	18.00
Induction coil, 1.2 cm. spark	8.50
Kundt's apparatus	4.00
Shellac bath-dish for chronograph	.50
Sonometer	6.00
Tuning fork, C = 256	1.00
Tuning fork, set of four, giving major chord	4.00
Velocity of sound in air apparatus	4.00

Light.

Candle	$ .05
Grating	$1.00 to 5.00
Lamp, oil	.30
Lens, convex, 3 in. diam., 25 cm. focus	.75
Lens, convex, 3 in. diam., 60 cm. focus	.75
Lens, concave, 3 in. diam.	.75
Mirror, plane, 3 × 4 in.	.20
Mirror, concave and convex, polished nickel	.50
Mirror, concave and convex, glass silvered, 4 in.	3.00
Optical bench, simple	1.50
Photometer disk	.20
Photometer bench	6.00
Porte lumière	$8.00 to 12.00
Prism, glass	.40
Refraction apparatus	.75
Spectrometer	40.00
Spyglass	2.00
Wooden screens, per set	.50

Heat.

Beaker, copper, 1 l.		$1.00
Beaker, copper, nickeled		.45
Beaker, glass, 1 l.		.24
Beaker, glass, ½ l.		.18
Beaker, tin		.20
Bunsen burner		.35
Coefficient of expansion of air		1.00
Coefficient of expansion of liquid		1.50
Coefficient of expansion of metal rod		2.50
Copper flask or boiler	$0.75 to	2.00
Iron stand, three rings		.65
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